MATHS – JEE ADVANCED PAPER – 1

SECTION – 1

1. Let M $=\begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$,

where $\alpha = \alpha(\theta) \, and \, \beta = \beta(\theta)$ are real number, and I is the 2 × 2 identity matrix. If

α* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi]\}$ and

β* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi]\}$

then the value of α* + β* is

(a) $-\frac{37}{16}$ (b) $-\frac{29}{16}$ (c) $-\frac{31}{16}$ (d) $-\frac{17}{16}$

Solution:

$$M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$$

$$M^2 = \alpha M + \beta I$$

$$a_{11} : \sin\theta - 1 - \sin^2\theta - \cos^2\theta - \cos^2\theta \sin^2\theta = \beta + \alpha \sin^4\theta$$

$$\sin^8\theta - 2 - \cos^2\theta \sin^2\theta = \beta + \alpha \sin^4\theta$$

$$a_{21} : \sin^4\theta + \cos^2\theta \sin^4\theta + \cos^4\theta + \cos^6\theta = \alpha\left(1+\cos^2\theta\right)$$

$$\left(1+\cos^2\theta\right)\alpha = \sin^4\theta\left(1+\cos^2\theta\right) + \cos^4\theta\left(1+\cos^2\theta\right)$$

$$\alpha = \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - \left(\frac{\sin^2\theta}{2}\right)$$

$$\alpha_{\min} = \frac{1}{2}$$

$$\beta = \cancel{\sin^8\theta} - \cos^2\theta \sin^2\theta - 2 - \cancel{\sin^2\theta} - \sin^4\theta \cos^4\theta$$

$$=-2-\left(\frac{\sin^2 2\theta}{4}\right)-\left(\frac{\sin^4 2\theta}{16}\right)$$

$$\beta_{\min}=-2-\frac{1}{16}\left(4t^2+t^4\right)$$

$$=-2-\frac{1}{16}\left(t^2+2\right)^2+\frac{1}{4}$$

$$=-\frac{7}{4}-\frac{1}{16}(9)=-\frac{37}{16}$$

$$\alpha+\beta=-\frac{37}{16}+\frac{1}{2}=-\frac{29}{16}$$

2. A line y = mx + 1 intersects the circle $(x-3)^2+(y+2)^2=25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct?

(a) $6\le m<8$ (b) $2\le m<4$ (c) $4\le m<6$ (d) $-3\le m<-1$

Solution:

$$y=mx+1$$

$$(x-3)^2+((mx+1)+2)^2=25$$

$$\Rightarrow x^2\left(1+m^2\right)+6(m-1)x-7=0$$

$$\frac{-3}{5}=\frac{\alpha+\beta}{2}=\frac{-6(m-1)}{2\left(1+m^2\right)}$$

$$1+m^2=5m-5$$

$$m^2-5m+6=0 \quad m=2,3$$

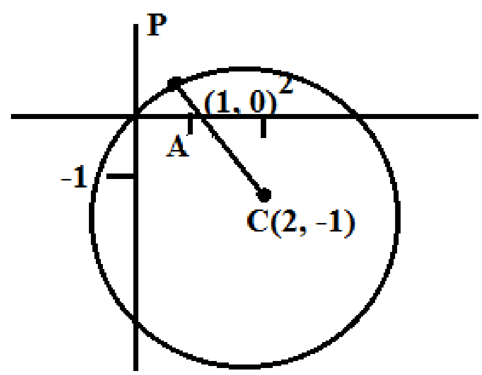
3. Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principle argument of $\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$ is

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

Solution:

$$|z-2+i| \geq \sqrt{5}$$

P is along $\overline{AC}$ but at $\sqrt{5}$ distance from C.



$$\overline{CP} = \sqrt{5}\frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \sqrt{5}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

$$\vec{P} = (2\hat{i}+\hat{j}) = \frac{-\sqrt{5}}{\sqrt{2}}(\hat{i}-\hat{j})$$

$$\vec{P} = \left(2-\frac{\sqrt{5}}{\sqrt{2}}\right)\hat{i} + \left(\frac{\sqrt{5}}{\sqrt{2}}-1\right)\hat{j}$$

$$z_0 = 2-\sqrt{\frac{5}{2}} + \left(4+\sqrt{\frac{5}{2}}\right)$$

$$\arg\left(\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}\right)$$

$$z_0+\bar{z}_0 = 4-\sqrt{10}$$

$$z_0-\bar{z}_0 = -2i+\sqrt{10}\,\hat{i}$$

$$\arg\left(\frac{\sqrt{10}}{\sqrt{10}\,i}\right)$$

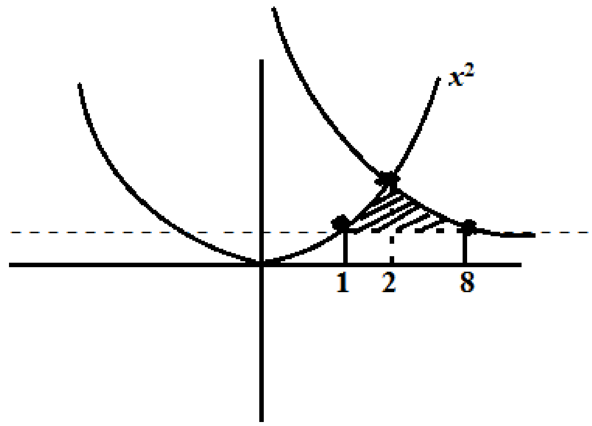
$$= -\frac{\pi}{2}$$

4. The area of the region $\left\{(x, y): xy \le 8, 1 \le y \le x^2\right\}$ is

(a) $8\log_e 2 - \frac{14}{3}$ (b) $16\log_e 2 - \frac{14}{3}$ (c) $16\log_e 2 - 6$ (d) $8\log_e 2 - \frac{7}{3}$

Solution:

$\left\{(x, y): xy \le 8, 1 \le y \le x^2\right\}$



$$\frac{8}{x} = x^2$$

$$\int_1^2 \left(x^2 - 1\right)dx + \int_2^8 \left(\frac{8}{x} - 1\right)dx$$

$$= 16\log_e 2 - \frac{14}{3}$$

SECTION – 2

1. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls, Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

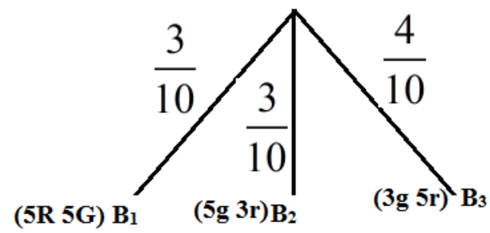
(a) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(b) Probability that the chosen ball is green equals $\frac{39}{80}$

(c) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(d) Probability that the selected bag is B_3, given that the chosen balls is green, equals $\frac{5}{13}$

Solution:



(i) $P(B_3 \cap G) = P\left(\frac{G}{B_3}\right).P(B_3)$

$$= \frac{3}{8} \times \frac{4}{10} = \frac{3}{20}$$

(ii) $P(G) = \frac{5}{10} \times \frac{3}{10} + \frac{5}{8} \times \frac{3}{10} + \frac{3}{8} \times \frac{4}{10}$

$$= \frac{60+75+60}{400} = \frac{195}{400} = \frac{39}{80}$$

(iii) $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

(iv) $P\left(\frac{B_3}{G}\right) = \frac{P(G \cap B_3)}{P(G)}$

$$= \frac{\frac{3}{20}}{\frac{39}{80}} = \frac{4}{13}$$

B, C

2. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and {R_1, R_2, R_3, …..} of rectangles as follows:

$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1;$

R_1: rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n: Ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in $R_{n-1}, n > 1;$

R_n: rectangle of largest area, with sides parallel to the axes, inscribed in E_n, n > 1.

Then which of the following options is/are correct?

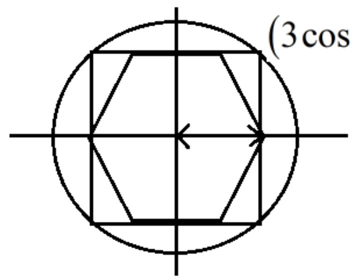
(a) The eccentricities of E_{18} and E_{19} are NOT equal

(b) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(c) The length of latus rectum of E_9 is $\frac{1}{6}$

(d) $\sum_{n=1}^{N}\left(area\, of\ R_n\right) < 24,$ for each positive integer N

Solution:



$$A = 6\cos\theta.4\sin\theta$$

= 12 sin 2 θ → max

$$\theta = \frac{\pi}{4}$$

$$E_2 = a_2 = 3\ \cos\frac{\pi}{4} = \frac{3}{\sqrt{2}}$$

$$b_2 = 2\sin\theta = \sqrt{2}$$

$$r = \frac{1}{\sqrt{2}}\quad a_n = \frac{3}{\left(\sqrt{2}\right)^{n-1}}\quad b_n = \frac{2}{\left(\sqrt{2}\right)^{n-1}}$$

$$e_{18} = 1 - \frac{b^2}{a^2} = 1 - \frac{4}{\cancel{\left(\sqrt{2}\right)^{2n-2}}}\left(\cancel{\sqrt{2}^{2n-2}}\right) = \frac{\sqrt{5}}{3}$$

e of all ellipses → same

Difference of *f* from conic in equation $a_q e$

$$E_a = \frac{2b_n^2}{a_n} = \frac{3}{\left(2\right)^8} \times \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{16}$$

$$= \frac{2\times 4}{4\left(\sqrt{2}\right)} = \frac{1}{6}$$

3. Let $M=\begin{bmatrix}0 & 1 & a\\ 1 & 2 & 3\\ 3 & b & 1\end{bmatrix}$ and adjM $=\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & 2\\ -5 & 3 & -1\end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct?

(a) a + b = 3

(b) $\det(\mathrm{adj}M^2) = 81$

(c) $(\mathrm{adj}M)^{-1} + \mathrm{adj}M^{-1} = -M$

(d) If M $\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

Solution:

$$M=\begin{bmatrix}0 & 1 & a\\ 1 & 2 & 3\\ 3 & b & 1\end{bmatrix} adjM=\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & 2\\ -5 & 3 & -1\end{bmatrix}$$

$$\Rightarrow adjM=\begin{bmatrix}2-3b & ab-1 & -1\\ 8 & -6 & 2\\ b-6 & 3 & -1\end{bmatrix}$$

$$b-6=-5$$

$$b=1$$

$$ab-1=1$$

$$a=2$$

$$M=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$$

$$|M|=-2$$

$$a+b=2$$

$$|adjM^2|=|M^2|^2=|M|^4=16$$

$\left(adjM\right)^{-1} + adjM^{-1}$

$\Rightarrow 2\left(adjM\right)^{-1}$

$= 2\left(M^{-1}\right)M$

$= 2\times\left(\dfrac{1}{-2}\right)M = -M$

$$M\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix} \Rightarrow \begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

$\beta + 2\gamma = 1$

$\alpha + 2\beta + 3\gamma = 2$

$3\alpha + \beta + \gamma = 1$ $\qquad\qquad$ $\alpha = 1 \quad \beta = -1 \quad \gamma = 1$

$\alpha - \beta + \gamma = 3$

4. Let $f : R \to R$ be given by

$$f\left(x\right) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \dfrac{2}{3}x^3 - 4x^2 + 7x - \dfrac{8}{3}, & 1 \le x < 3; \\ \left(x-2\right)\log_e\left(x-2\right) - x + \dfrac{10}{3}, & x \ge 3 \end{cases}$$

Then which of the following options is/are correct?

(a) f' has a local maximum at x = 1 $\qquad$ (b) f is onto

(c) f is increasing on $(-\infty, 0)$ $\qquad$ (d) f' is NOT differentiable at x = 1

Solution:

$$f(x)=\begin{cases}(a+1)^5-2a & x<0\\ x^2-x+1 & 0\le x<1\\ \frac{2}{3}x^3-4x^2+7x-\frac{8}{3} & 1\le x<3\\ (x-2)\log_e(x-2)-x+\frac{10}{3} & x\ge 3\end{cases}$$

When x < 0, $f\rightarrow$ unit

$f'(x)=5(x+1)^4-2$ can change sing for x < 0

Range : -∞ + 1

∴ Not monotonic

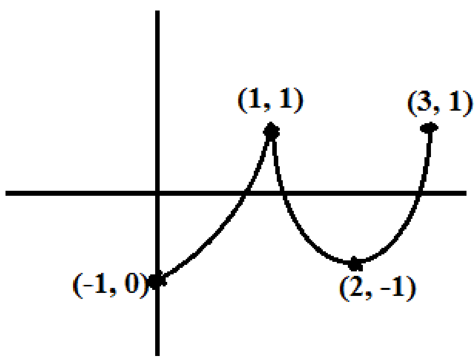
$$\frac{f(x)=x^2-x+1}{f'(x)=2x-1}$$

Max at x = 0 & 1

$x\ge 3$

$f(3)=1\log 1-3+\frac{10}{3}=\frac{1}{3}$

$f(\infty)\rightarrow\infty$



$$f'(x):\begin{cases}2x-1 & (0\le x<1)\\ 2x^2-8x+7 & |\le x<3\end{cases}$$

Loc. Max at x = 1

5. Let α and β be the roots of $x^2-x-1=0,$ with α > β. For all positive integers n, define

$$a_n=\frac{\alpha^n-\beta^n}{\alpha-\beta}, n\ge 1$$

$$b_1=1 \text{ and } b_n=a_{n-1}+a_{n+1}, n\ge 2.$$

Then which of the following options is/are correct?

(a) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \ge 1$

(b) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(c) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(d) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$

Solution:

$$x^2 - x - 1 = 0$$

$$\alpha = \frac{1+\sqrt{5}}{2} \quad \beta = \frac{1-\sqrt{5}}{2}$$

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \qquad b_1 = 1$$

$$b_n = a_{n-1} + a_{n+1};\ n \ge 2$$

$$a_{n+2} - a_{n+1} = \left(\frac{\alpha^{n+2} - \beta^{n+2}}{\alpha - \beta}\right) - \left(\frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}\right)$$

$$= \frac{\alpha^n\left(\alpha^2 - \alpha\right) - \beta^n\left(\beta^2 - \beta\right)}{\alpha - \beta}$$

$$= \frac{\alpha^n(1) - \beta^n(1)}{\alpha - \beta} = a_n$$

$$a_1 + a_2 + ... + a_n \qquad \Rightarrow a_n + a_{n+1} = a_{n+2}$$

$$\sum_{n=1}^{\infty} \frac{a_n}{10^n}$$

$$\Rightarrow \frac{\sum\left(\frac{\alpha}{10}\right)^n - \sum\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\sum_{r=1}^{n} a_r = a_{n+2} - a_2$$

$$= a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$$= a_{n+2} - (\alpha + \beta)$$

$$= a_{n+2} - 1$$

$$\Rightarrow \frac{\dfrac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1-\frac{\beta}{10}}}{\alpha - \beta} = \frac{\dfrac{\alpha}{10-\alpha} - \dfrac{\beta}{10-\beta}}{\alpha - \beta} = \frac{10}{(10-\alpha)(10-\beta)} = \frac{10}{89}$$

$$\sum \frac{b_n}{10^n} = \sum \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{12}{89}$$

$$b_n = a_{n-1} + a_n$$

$$= \frac{\left(\alpha^{n-1} - \beta^{n-1}\right) + \left(\alpha^{n+1} - \beta^{n+1}\right)}{\alpha - \beta}$$

$$= \alpha\beta = -1 \quad \alpha^{n-1} = -\alpha^n \beta$$

$$\Rightarrow \frac{-\alpha^n \beta + \beta^n \alpha + \alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$$

$$= \frac{\alpha^n (\alpha - \beta) + \beta^n (\alpha - \beta)}{\alpha - \beta} = \alpha^n + \beta^n$$

6. Let Γ denote a curve y = y(x) which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the y-axis at Y_P. If PY_P has length 1 for each point P on Γ, then which of the following is options is/are correct?

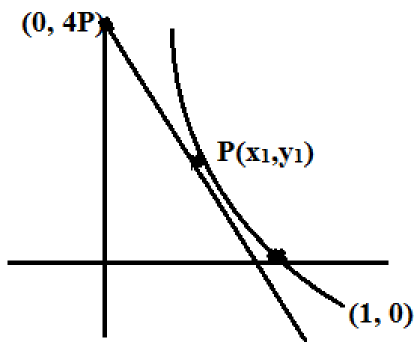
(a) $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$ (b) $xy' - \sqrt{1-x^2} = 0$

(c) $y - \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(d) $xy' + \sqrt{1-x^2} = 0$

Solution:

(1, 0)



$$y - y_1 = m\left(x - x_1\right)$$

$$y_P - y_1 = -mx_1$$

$$y_P = mx_1 + y_1$$

$$\Rightarrow 1 = x_1^2 + m^2x_1^2$$

$$1 = x^2\left(1 + \left(\frac{dy}{dx}\right)^2\right)$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{x^2} - 1$$

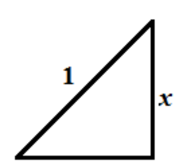
$$\frac{dy}{dx} = \pm\frac{\sqrt{1-x^2}}{x}$$

$$y = \pm\int\frac{\sqrt{1-x^2}}{x}dx$$

$$x\sin\theta \quad dx = \cos\theta d\theta$$

$$= \pm\int\frac{\cos^2\theta\, d\theta}{\sin\theta}$$

$$= \pm\int\left(\cos ec\theta - \sin\theta\right)d\theta$$



$$= \pm\log\left|\left(\cos ec\theta + \cot\theta\right)\right| + \cos\theta$$

$$= \pm \log\left|\frac{1}{x} + \sqrt{1-x^2}\right| + \left(\sin^{-1} x\right) + \sqrt{1-x^2} + c$$

$$0 = \pm \log(1) + \sqrt{1-1} + C$$

$$C = 0 \qquad\qquad \Rightarrow B, D$$

A, B, C, D

7. In a non-right-angle triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at 0. If $p = \sqrt{3},\ q = 1,$ and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

(a) Area of ΔSOE $= \dfrac{\sqrt{3}}{12}$

(b) Radius of incircle of ΔPQR $= \dfrac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(c) Length of RS $= \dfrac{\sqrt{7}}{2}$

(d) Length of OE $= \dfrac{1}{6}$

Solution:

$$\frac{\sin P}{\sqrt{3}} = \frac{\sin Q}{1} = \frac{1}{2R} = \frac{1}{2}$$

$$\angle P = \frac{\pi}{3} (or) \frac{2\pi}{3} \quad \angle Q = \frac{\pi}{6} (or) \frac{5\pi}{6}$$

$$p > q \Rightarrow \angle P > \angle Q$$

If $\angle P = \dfrac{\pi}{3}$ & $\angle Q = \dfrac{\pi}{6} \Rightarrow R = \dfrac{\pi}{2}$ (not possible)

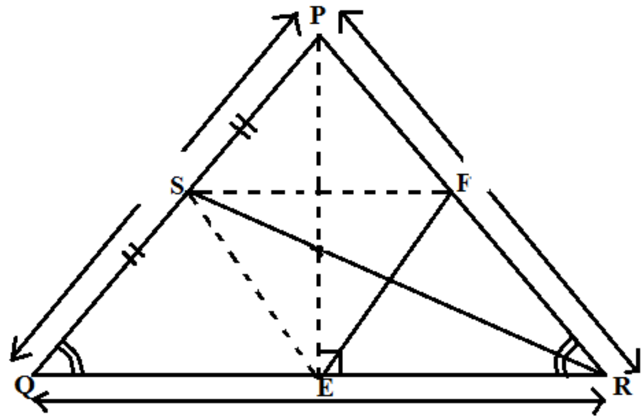
$$\therefore \angle P = \frac{2\pi}{3} \ \& \ \angle Q = \angle R = \frac{\pi}{6}$$

$$r = \frac{\Delta}{S} = \frac{\frac{1}{2}.1.\sqrt{3} = \frac{1}{2}}{\left(\frac{\sqrt{3}+2}{2}\right)} = \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$$

$$\Delta SEF = \frac{1}{4} ar(\Delta PQR)$$

$$ar(\Delta SOE) = \frac{1}{3} ar(\Delta SEF) = \frac{1}{12} ar(\Delta PQR)$$

$$= \frac{1}{12} \cdot \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{48}$$



$$RS = \frac{1}{2}\sqrt{6+2-1} = \frac{\sqrt{7}}{2}$$

$$OE = \frac{1}{3} PE = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\sqrt{2+2-3} = \frac{1}{6}$$

8. Let L_1 and L_2 denotes the lines

$$\vec{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right), \lambda \in \mathbb{R}$$

$$\vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in \mathbb{R}$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

(a) $\vec{r} = \frac{1}{3}\left(2\hat{i} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

(b) $\vec{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

(c) $\vec{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

(d) $\vec{r} = \frac{2}{9}\left(4\hat{i} + \hat{j} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

<u>Solution:</u>

$L_1 : \vec{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$L_2 : \vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

$$L_1 : \frac{x-1}{-1} = \frac{y-0}{2} = \frac{z-0}{2}$$

$L_2 : \frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$

$L_3 : \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

$L_3 : L_1 \times L_2$

ll^{ly} $6\hat{i} + 6\hat{j} - 3\hat{k}$

A on $L_1 \left(-\lambda + 1, 2\lambda, 2\lambda\right)$

B on $L_2 : \left(2\mu, -\mu, 2\mu\right)$

AB Δrs: $\left(2\mu + \lambda - 1, -\mu - 2\lambda, 2\mu - 2\lambda\right)$

ΔR of AB: (6, 6, -3) or (2, 2, -1)

$\Rightarrow \frac{2\mu + \lambda - 1}{2} = \frac{-\mu - 2\lambda}{2} = \frac{2\mu - 2\lambda}{-1} = k$

$\lambda = \frac{3k+1}{3} \quad \mu = -4k - \frac{2}{3}$

$= k + \frac{1}{3}$

$= 2\mu - 2\lambda + k = 0$

$\Rightarrow 2\left(4k - \frac{2}{3}\right) - 2\left(\frac{3k}{3} + 1\right) + k = 0$

$\Rightarrow k = \frac{-2}{9} \Rightarrow \lambda = \frac{1}{9}, \mu = \frac{2}{9}$

A: $\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$ $B : \left(\frac{4}{9}, \frac{-2}{9}, \frac{4}{9}\right)$

Mid point : $\left(\frac{2}{3}, 0, \frac{1}{3}\right)$

SECTION – 3

1. If $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$

Then $27I^2$ equals __________

Solution:

$$I = \frac{2}{\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$$

$$I = \frac{2}{\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(1+e^{-\sin x}\right)\left(2-\cos 2x\right)}$$

$$\cancel{2}I = \frac{\cancel{2}}{\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{\cancel{\left(1+e^{\sin x}\right)}dx}{\cancel{\left(1+e^{\sin x}\right)}\left(2-\cos 2a\right)}$$

$$= \frac{2}{\pi}\int_{0}^{\frac{\pi}{4}} \frac{dx}{1+2\sin^2 x} = \frac{2}{\pi}\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x\, dx}{3\tan^2 x+1} = \frac{2}{3\sqrt{3}}$$

Ans: 4

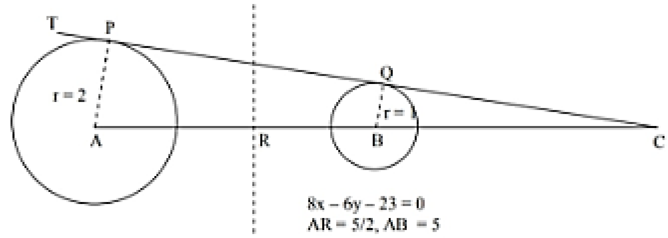
2. Let the point B be the reflection of the point A(2, 3) with respect to the line 8x – 6y – 23 = 0. Let Γ_A and Γ_B be circle of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circle Γ_A and Γ_B such that both the circle are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ________

Solution:

Now ΔAPC and BQC are similarly

$$\frac{BC}{AC} = \frac{1}{2} \Rightarrow 2(AC - AB) = AC$$

AC = 2AB = 10



3. Let AP (a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If AP(1; 3) ∩ AP (2; 5) ∩ AP (3; 7) = AP (a; d) then a + d equals)_________

Solution:

$$\alpha, \alpha + d, \ldots . \alpha > 0$$

I: 1, 4, 7, 10, 13, 16 ….. 32

II: 2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52

III: 3, 10, 17, 24, 31, 38, 45, 52

52 ← a + d → LCM of 3, 5, 7 = 105

a + d = 157

4. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

   E_1 = {A ϵ S : det A = 0} and

   E_2 = {A ϵ S : sum of entries of A is 7}.

   If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals …….

Solution:

S : 2^9

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

E_2 : Sum of entries 7

7 1s 2 0's

Total $E_2 = \frac{9!}{7!2!} = 36$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

Per |A| to be 0, both zeroes should be in the same row/column.

$\therefore\ 3\times3\times2 = 18$ cases

$$P\left(\frac{E_1}{E_2}\right) = \frac{18}{36} = \frac{1}{2}$$

5. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in \mathbb{R}$

$\vec{r} = \mu\left(\hat{i} + \hat{j}\right), \mu \in \mathbb{R}$ and

$\vec{r} = v\left(\hat{i} + \hat{j} + \hat{k}\right), v \in \mathbb{R}.$

Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals __________

Solution:

$\vec{r} = \lambda\hat{i}, \lambda \in \mathbb{R}$

$\vec{r} = \mu\left(\hat{i} + \hat{j}\right), \mu \in \mathbb{R}$ cuts $x + y + z = 1$ at A, B, C graph

$\vec{r} = v\left(\hat{i} + \hat{j} + \hat{k}\right), v \in \mathbb{R}.$

1st line : $x = \lambda, y = 0, z = 0$

$x + y + z = 1 \Rightarrow \lambda = 1\ \ A(1,0,0)$

2nd line: $x = \mu\ \ y = \mu\ \ z = 0$

$$\therefore 2\mu = 1 \;\; \mu = \frac{1}{2} \;\; B\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

Parallels C: $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$= \frac{1}{2}\left|\frac{\hat{i}}{6} + \frac{\hat{j}}{5} + \frac{\hat{k}}{6}\right| = \frac{\sqrt{3}}{12} \qquad (6\Delta) = \frac{3}{6}$$

6. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$$\left\{\left|a + b\omega + c\omega^2\right|^2 : \text{a,b,c distinct non-zero integers}\right\}$$

equals ________

Solution:

$$\left|a + b\omega + c\omega^2\right|^2$$

$$= \left(a + b\omega + c\omega^2\right)\left(a + b\omega^2 + c\omega\right)$$

$$= \left(a^2 + b^2 + c^2 = ab - bc - ca\right) \qquad \left[\because\, a + b\omega + c\omega^2 = \overline{a} + \overline{b\omega} + \overline{c\omega^2} = a + b\omega^2 + c\omega\right]$$

$$\Rightarrow \frac{1}{2}\left((a-b)^2 + (b-c) + (c-a)^2\right)$$

$$= \frac{1}{2}(1 + 1 + 4) = 3$$

PHYSICS – JEE ADVANCED PAPER – 1

SECTION – 1

1. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T(t) = T_0 (1 + \beta t^{1/4})$ where β is a constant with appropriate dimension of temperature. The heat capacity of metal is:

(a) $\frac{4P\left(T(t)-T_0\right)^3}{\beta^4 T_0^4}$ (b) $\frac{4P\left(T(t)-T_0\right)^2}{\beta^4 T_0^3}$ (c) $\frac{4P\left(T(t)-T_0\right)^4}{\beta^4 T_0^5}$ (d) $\frac{4P\left(T(t)-T_0\right)}{\beta^4 T_0^2}$

Solution:

$$\text{Heat capacity } = \frac{dQ}{dT}$$

$$H = \frac{dQ}{dT} \Rightarrow \frac{dQ}{dt} = H.\frac{dT}{dt}$$

$$P = H\frac{d}{dt}T$$

$$= \frac{H\,T_0}{4}\beta \quad t^{-3/4}$$

$$\frac{4P}{T_0\beta} = H\ t^{-3/4}$$

$$t^{-3/4} = \left(\frac{T-T_0}{T_0\beta}\right)^3$$

$$H = \frac{4P\left(T-T_0\right)^3}{T_0^4\beta^4}$$

2. In a capillary tube of radius 0.2 mm the water rises up to height of 7.5 cm with angle of contact equal to zero. If another capillary with same radius but of different material dipped in the same liquid. The height of water raised in capillary will be, if angle of contact becomes 60°.

(a) 7.5 cm (b) 15 cm (c) 3.75 cm (d) 30 cm

Solution:

$$T = \frac{Rh\rho g}{2\cos\theta}$$

$$\frac{h}{\cos\theta} = const$$

$$\frac{7.5}{1} = \frac{h'}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow h' = 3.75cm$$

3. A sample of ${}_{19}K^{40}$ disintegrates into two nuclei Ca & Ar with decay constant $\lambda_{Ca} = 4.5\times10^{-10}S^{-1}$ and $\lambda_{Ar} = 0.5\times10^{-10}S^{-1}$ respectively. The time after which 99% of ${}_{19}K^{40}$ gets decayed is:

(a) 6.2×10^9 sec (b) 9.2×10^9 sec (c) 7.2×10^9 sec (d) 4.2×10^9 sec

Solution:

$$\lambda = \lambda$$

$$\frac{1}{100}\cancel{No} = \cancel{No}\ e^{-\lambda t}$$

$$\ln\left(\frac{1}{100}\right) = -\left(\lambda_1 + \lambda_2\right)t + \ln 100 = +\left(\lambda_1 + \lambda_2\right)t^2$$

$$\frac{2.303\times2}{5\times10^{-10}} = t$$

$$t = 9.2\times10^9\,\text{sec}$$

4. Consider a spherical gaseous cloud of mass density ρ(r) in a free space where r is the radial distance from its centre. The gaseous cloud is made of particles of equal mass m moving in circular orbits about their common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If ρ(r) is constant with time. The particle number density n(r) = ρ(r)/m is:

(g = universal gravitational constant)

(a) $\frac{3K}{\pi r^2 m^2 G}$ (b) $\frac{K}{2\pi r^2 m^2 G}$ (c) $\frac{K}{\pi r^2 m^2 G}$ (d) $\frac{K}{6\pi r^2 m^2 G}$

Solution:

$$\frac{GMm}{r} = \frac{m\upsilon^2}{r}$$

$$= \frac{2}{r}\frac{1}{2}m\upsilon^2$$

$$=\frac{2}{r}k$$

$$M=\frac{2kr}{Gm}$$

$$4\pi r^2 \cancel{dr}\,\rho=\frac{2k\cancel{dr}}{Gm}$$

$$\rho=\frac{k}{2\pi Gmr^2}$$

SECTION – 2

5. A thin spherical insulating shell of radius R caries a uniformly distributed charge such that the potential at its surface is V_0. A hole with small area $\alpha 4\pi R^2\left(\alpha<<1\right)$ is made in the shell without effecting the rest of the shell. Which one of the following is correct.

(a) The magnitude of $\vec{E}$ at a point located on a line passing through the hole and shell's centre on a distance 2R from the centre of spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

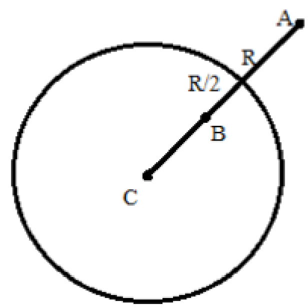
(b) Potential at the centre of shell is reduced by $2\alpha V_0$.

(c) The magnitude of $\vec{E}$ at the centre of shell reduced by $\frac{\alpha V_0}{2R}$

(d) The ratio of potential at the centre of the shell to that of the point at $\frac{1}{2}$ R from centre towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

Solution:

$$dq=\frac{Q}{4\pi R^2}-dA=Q\alpha$$



$$V_C=\frac{KQ}{R}-\frac{K\alpha Q}{R}=V_0\left(1-\alpha\right)$$

$$V_B=\frac{KQ}{R}-\frac{K\alpha Q}{R/2}=V_0\left(1-2\alpha\right)$$

$$V_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$$

(reduced by $\frac{\alpha V_0}{R}$)

$$E @ C = \frac{K\alpha Q}{R^2} = \frac{\alpha V_0}{R}$$

Increased by $\frac{2V_0}{R}$

6. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r & with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top & bottom surfaces. Which of the following are correct.

(a) If h > 2R & r > R then $\phi = \frac{Q}{\varepsilon_0}$

(b) If $h < \frac{8R}{5} \,\&\, r = \frac{3R}{5}$ then $\phi = 0$

(c) If h > 2R & r $= \frac{4R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

(d) If h > 2R & r $= \frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

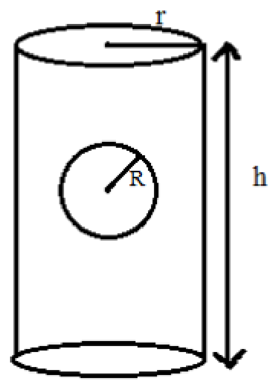
Solution:

(1) If h > 2R

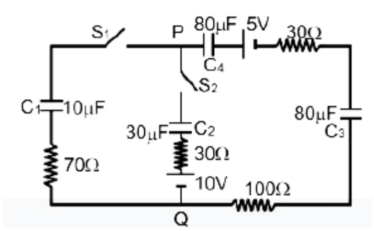
$$Q = \frac{Q}{E_0}$$

(2) $h = \frac{8R}{5}$ $\quad r = \frac{3R}{5}$

Using Gauss law concept ABD are correct



7. Which statements is/are correct:



(a) At time t = 0, the S_1 is closed instantaneous current in the closed circuit will be 25 mA

(b) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed at this time the instantaneous current across 30Ω resistor between P & Q will be 0.2A.

(c) If key S_1 is kept closed for long time such that capacitors are fully charged the voltage across C_1 will be 4V.

(d) If S_1 is kept closed for long time such that capacitors are fully charged the voltage difference between P & Q will be 10V.

8. A galvanometer of resistance 10 ohm and maximum current of 2μA is converted into voltmeter of range 100mV and when converted into ammeter then range is 1mA. When these voltmeter and ammeter are connected by a (ideal) battery in series with a resistance of R = 1000Ω, then

(a) Measured value of R is between 978Ω and 996Ω

(b) Resistance of voltmeter $10^5 \Omega$

(c) Shunt resistance is 20mΩ

(d) If the ideal battery is replaced by non-ideal battery with internal resistance of 5Ω then R will be > 1000 Ω

Solution:

$$V = 100 \times 10^{-3} V$$

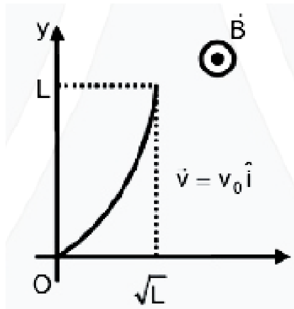
$$V = Ig\left(Rg + R\right)$$

$$= \frac{10^{-1}}{2 \times 10^{-6}} = \left(Rg + R\right)^{R_V}$$

$$R_V = 5 \times 10^4$$

$$S = \left(\frac{10}{\frac{10^{-3}}{2 \times 10^{-3}} - 1} \right) = 20\, m\Omega$$

9. Conducting wire of parabolic shape, initially $y = x^2$ is moving with velocity $\vec{v} = v_0 \hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0 \left(1 + \left(\frac{y}{L} \right)^{\beta} \right) \hat{k}$ as shown in figure. If V_0, B_0, L & B are +ve constants & Δϕ is potential difference develop between the ends of wire, then correct statement(s) is/are



(a) $|\Delta\phi| = \frac{1}{2} B_0 V_0 L$ for β = 0

(b) $|\Delta\phi| = \frac{4}{3} B_0 V_0 L$ for β = 2

(c) $|\Delta\phi|$ is proportional to the length of wire projected on y-axis

(d) $|\Delta\phi|$ remains same if the parabolic wire is replaced by a straight wire, y = x, initially of length $\sqrt{2}\ell$

Solution:

$$d\varepsilon = BV_0 dy$$

$$= B_0 \left\{1 + \left(\frac{y}{2}\right)^{\beta}\right\} V_0\, dy$$

$$\varepsilon = B_0 \int_0^L 4 + \left(\frac{y}{L}\right)^{\beta} \Bigg\} V_0\, dy$$

$$= B_0\ V_0 L \left(1 + \frac{1}{\beta + 1}\right)$$

Let $\beta = 0 \quad \varepsilon = 2B_0 V_0 L$

$$\beta = 2 \quad \varepsilon = B_0 V_0 L \left(1 + \frac{1}{3}\right)$$

$$= \frac{4}{3} B_0 V_0 L$$

B, C is correct

D is also correct because projection of wire on y axis is same

10. If in a hypothetical system if the angular momentum and mass are dimensionless. Then which of the following is true.

(a) The linear momentum varies as L^{-1} (b) The energy varies as L^{-2}

(c) The power varies as L^{-4} (d) The force varies as L^{-5}

Solution:

$$[M] = \left[M^0 L^0 T^0\right]$$

$$[J] = \left[ML^2T^{-1}\right]$$

$$\left[ML^2T^{-1}\right] = \left[M^0 L^0 T^0\right]$$

$$\Rightarrow \left[L^2\right] = [T]$$

Momentum

$$[P] = \left[MLT^{-2}.LT^{-1}\right]$$

$$= \left[ML^2T^{-3}\right]$$

$$= \left[L^2 L^{-6}\right]$$

$$= \left[L^{-4}\right]$$

$$[E] = \left[MLT^{-2}.L\right]$$

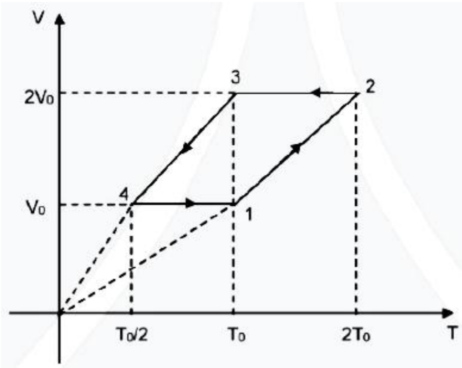
$$= L^2 \;\; L^{-4}$$

$$= \left[L^{-2}\right]$$

$$[F] = \left[L \;\; L^{-4}\right] = \left[L^{-3}\right]$$

A, B, C

11. V – T diagram for n mol monoatomic gas is given below:



Choose the correct statement:

(a) $\left|\frac{\Delta Q_{1\to2}}{\Delta Q_{3\to4}}\right| = \frac{1}{2}$

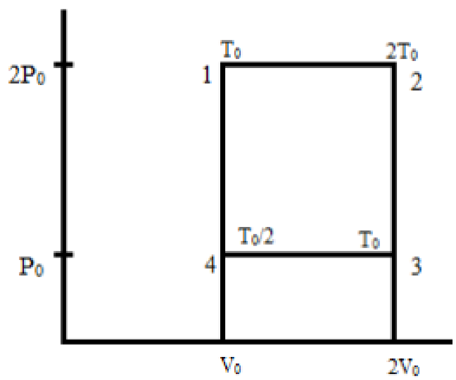
(b) $\left|\frac{\Delta Q_{1\to2}}{\Delta Q_{2\to3}}\right| = \frac{5}{3}$

(c) Work done in cyclic process is $\Delta W = \frac{nRT_0}{2}$

(d) There are only adiabatic and isochoric processes are involved.

Solution:

Corresponding PV entraps



(A) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{3\to4}}\right| = \left[\frac{NC_P\Delta T_{1\to2}}{NC_P\Delta T_{3-4}}\right] =$

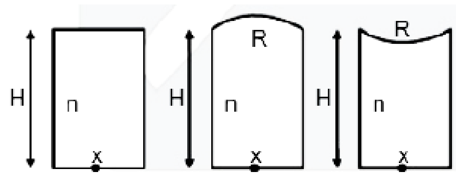
$$= \frac{T_0}{T_0/2} = 2$$

(B) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{2\to3}}\right| = \left|\frac{NC_P\Delta T_{1\to2}}{NC_V\Delta T_{2\to3}}\right|$

$$= \frac{C_P}{C_V} = \frac{5}{3}$$

(C) $\omega = amu = P_0V_0$

12. Apparent depth for point object x in all three cases are H_1, H_2 & H_3 respectively when seen from below given H = 30 cm, n = 1.5 & R = 3m, then



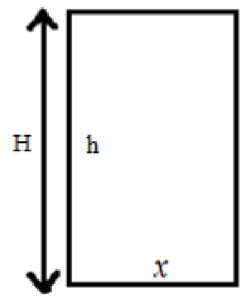
(a) $0.8 < H_2 - H_1 < 0.9$ (b) $H_2 > H_1$ (c) $H_2 > H_3$ (d) $H_3 > H_1$

Solution:

Case I

$$\frac{n_2}{n_1} = \frac{d^1}{d} \Rightarrow \frac{1}{n} = \frac{d^1}{(3Q)}$$

$$d^1 = \frac{30}{3} \times 2 = 20cm$$

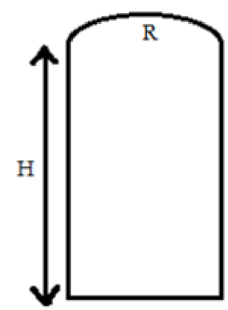


Case II

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{1^2}$$

$$\frac{1}{-v_2} - \frac{3}{-2 \times 30} = \frac{1 - 3/2}{-300}$$

$$H_2 = 20.684$$



Case II

$$V_3 = 19.354$$

SECTION – 3

13. Consider the following nuclear fission reaction

$${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4} + Q.$$

In this fission reaction. Kinetic energy of α-particle emitted is 4.44 MeV. Find the energy emitted as γ – radiation in keV in this reaction.

$$m\left({}_{88}Ra^{226}\right) = 226.005\,amu$$

$$m\left({}_{86}Rn^{222}\right) = 222.000\,amu$$

$$m\left({}_{2}He^{4}\right) = 4.000\,amu$$

Solution:

Δm = 0.005 amu

$$\frac{K_\alpha}{K_{Rn}} = \frac{m_{Rn}}{m_\alpha}$$

$$K_{Rn} = \frac{m_\alpha}{m_{Rn}} K_\alpha =$$

$$= \frac{4}{222} \times 4.44 = 0.08\,MeV$$

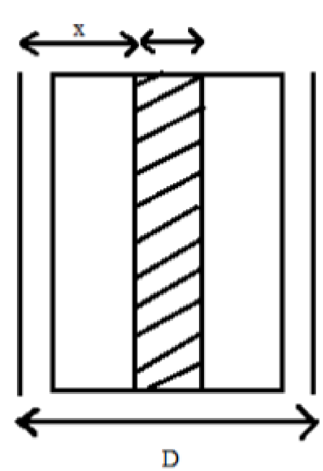
Energy of γ – photon

= 4.655 – (4.44 + 0.08)

= 0.135 MeV

14. N dielectrics are introduced in series in a capacitor of thickness D. Each dielectric have width d = D/N & dielectric constant of mth dielectric is given by $K_m = K\left(1 + m/N\right)$: [N >> 10^3, Area of plates = A] Net capacitance is given by $\frac{K\varepsilon_0 A}{\alpha D \ell n 2}$. Find value of α.

Solution:



$$\frac{x}{m} = \frac{\Delta}{N}$$

$$d\frac{1}{C} = \frac{dx}{Km\varepsilon_0 A} = \frac{dx}{K\varepsilon_0 A\left(H\frac{m}{N}\right)}$$

$$\int d\frac{1}{C} = \int_0^D \frac{dx}{K\varepsilon_0 A\left(H\frac{x}{D}\right)}$$

Integrating we get

$$C_{eq} = \frac{K\varepsilon_0 A}{D\ell n 2}$$

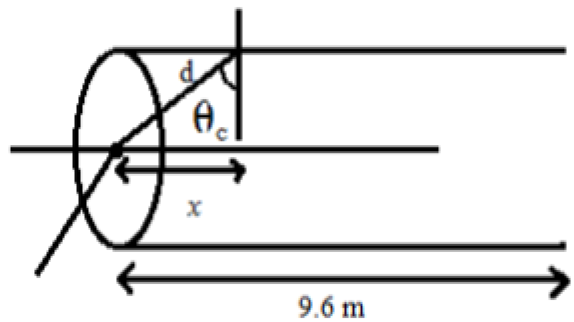
$\therefore\ \alpha = 1$

15. If at angle θ the light takes maximum time to travel in optical fiber. Then the maximum time is $x \times 10^{-8}$, calculate x.

Solution:

$$1.5\sin\theta_C = 1.44\sin 90°$$

$$\sin\theta_C = \frac{1.44}{1.5} = \frac{24}{25}$$



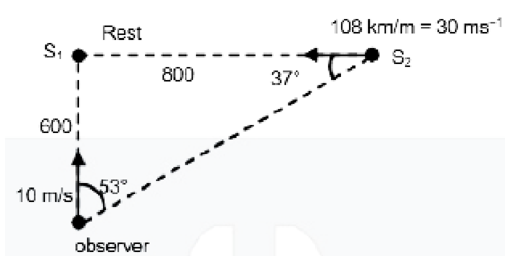
$$\sin\theta_C = \frac{x}{d}$$

$$\Rightarrow d = \frac{25x}{24}$$

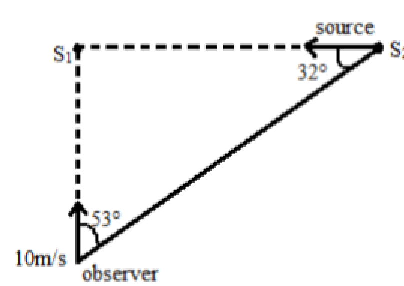
$$t = \frac{L}{\left(\frac{C}{n_2}\right)} = \frac{10}{\frac{3\times10^8}{1.5}}$$

$$t = 5\times10^8 s$$

16. The source S_1 is at rest. The observer and the source S_2 are moving towards S_1 as shown in figure. The roof beats observed by the observer if both sources have frequency 120 Hz and speed of sound 330 m/s in is



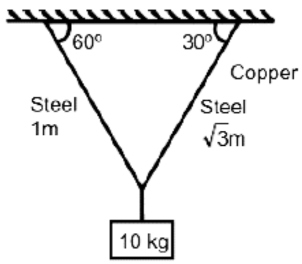
Solution:



$$f_b = 120\left(\frac{330+10\cos 53}{330-30\cos 37}\right)-\left(\frac{330+10}{330}\right)$$

= 8.128 Hz

17. A weight of 100 N is suspended by two wires made by steel and copper as shown in figure length of steel wire is 1 m and copper wire is $\sqrt{3}m.$ Find ratio of change in length of copper wire $(\Delta\ell_o)$ to change in length of steel wire $(\Delta\ell_s)$. Given Young's modulus: $Y_{steel} = 2 \times 10^{11}$ N/m^2, $Y_{copper} = 1 \times 10^{11}$ N/m^2.



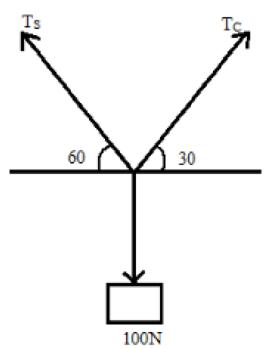
Solution:

$$\frac{T_S}{Z} = T_C\frac{\sqrt{3}}{2}$$

$$T_S = \sqrt{3}\ T_C$$

$$\frac{\Delta\lambda_C}{\Delta\lambda_S} = \left(\frac{T_C}{T_S}\right)\left(\frac{L_C}{L_S}\right)\frac{Y_S}{Y_C}$$

$$= \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{1}\right)\frac{2\times10^{11}}{1\times10^{11}} = 2$$



18. An optical bench, to measure the focal length of lens, is 1.5 m long and on the bench marks are with spacing $\frac{1}{4}$ cm. Now a lens is placed at 75 cm and pin type object is placed at 45 cm marks on the bench. If its image is formed at 135 cm find maximum possible error in calculation of focal length.

Solution:

V = 30 cm dv = 0.5 cm

V = 60 cm dv = 0.5 cm

$$\frac{1}{y}-\frac{1}{u}=\frac{1}{f}\Rightarrow f=20cm$$

$$\frac{-dv}{v^2}+\frac{-dv}{u^2}=\frac{-df}{f^2}$$

$$\frac{df}{f}\times 100=f\left[\frac{dv}{v^2}+\frac{du}{u^2}\right]$$

= 1.38 and 1.39

CHEMISTRY – JEE ADVANCED PAPER – 1

SECTION – 1

1. Which of the following set represent correct formula for Malachite, Magnetite, Calamine & Cryolite?

(a) $CuCO_3, Fe_2O_3, ZnO, Al_2O_3$ (b) $CuCO_3, Cu(OH)_2, Fe_3O_4, ZnCO_3, Na_3AlF_6$

(c) $CuCO_3, Fe_3O_4, ZnCO_3, Al_2O_3$ (d) $CuCO_3.Cu(OH)_2, Fe_2O_3, ZnCO_3, Na_3AlF_6$

Solution:

(B)

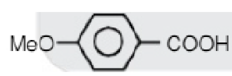
Malachite $\rightarrow$ $CuCO_3.Cu(OH)_2$

Magnetite $\rightarrow$ Fe_3O_4

Calamine $\rightarrow$ $ZnCO_3$

Cryollite $\rightarrow$ Na_3AlF_6

2. Find the correct acidic strength order:

(i) HC = C – COOH (ii) H_2C = CH – COOH (iii) 

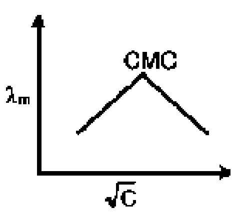
(iv) CH_3 – CH_2 – COOH

(a) i > ii > iv > iii (b) i > ii > iii > iv (c) iii > ii > i > iv (d) iii > i > iv > ii

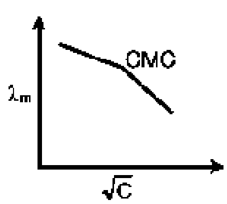
Solution:

(B)

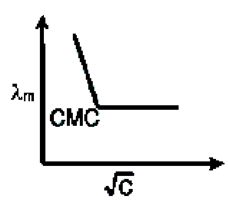
3. Sodium stearate is a strong electrolyte. Which of the following plot is correct regarding its conductance:



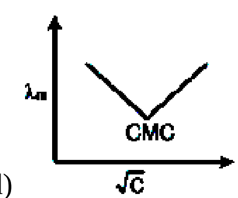
(a)



(b)



(c)



(d)

Solution:

(B)

By definition, $\lambda m \alpha \frac{1}{\sqrt{C}}$

4. Which green coloured compound of chromium is formed in borax bead test?

(a) $Cr(BO_2)_3$ (b) Cr_2O_3 (c) CrB (d) $CrBO_3$

Solution:

(A)

$$Na_2B_4O_7.10H_2O \xrightarrow{\Delta} Na_2B_4O_7$$

$$Na_2B_4O_7 \xrightarrow{\Delta} NaBO_2 + B_2O_3$$

$$Cr_2O_3 + B_2O_3 \longrightarrow \underset{green}{Cr(BO_2)_3}$$

SECTION – 2

5. Choose the reaction, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation:

(a) $2C_{(g)} + 3H_{2(g)} \rightarrow C_2H_{6(g)}$ (b) $\frac{3}{2}O_{2(g)} \rightarrow O_{3(g)}$

(c) $\frac{1}{8}S_{8(s)} + O_{2(g)} \rightarrow SO_{2(g)}$ (d) $2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O(\ell)$

Solution:

(B, C)

By definition,

Enthalpy of formation is defined as the Enthalpy change occurring when, a compound is formed from its constituent elements in standard state.

6. A Tin – chloride ‘P’ gives following reaction (unbalanced reaction)

$P + Cl^- \longrightarrow X$ [Monoanion pyramidal geometry]

$P + Me_3N \longrightarrow Y$

$P + CuCl_2 \longrightarrow Z + CuCl$

Then which of the following is/are correct.

(a) Y contains co-ordinate bond (b) X is sp^3 hybridised.

(c) Oxidation state of Sn is X is +1. (d) X contain lone pair on central atom.

Solution:

(A, B, D)

$$\underset{(P)}{SnCl_2} + Cl^- \longrightarrow \underset{(X)}{SnCl_3^-}$$

$$\underset{(P)}{SnCl_2} + Me_3N \longrightarrow \underset{(Y)}{SnCl_2[N\,Me_3]}$$

$$\underset{(P)}{SnCl_2} + CuCl_2 \longrightarrow \underset{(Z)}{SnCl_2} + CuCl$$

7. $^{238}_{92}U \xrightarrow{x_1} {}^{234}_{90}Th \xrightarrow{x_2} {}^{234}_{91}Pa \xrightarrow{x_3} {}^{234}Z \xrightarrow{x_4} {}^{230}_{90}Th$

$x_1, x_2, x_3, x_4,$ are either particles or radiation. Then

(a) x_1 is deflected toward negatively charged plate.

(b) x_2 is β-particle.

(c) x_3 is γ-radiation.

(d) z is isotope of ^{238}U

Solution:

(A, B, D)

$X_1 \rightarrow \alpha$ – decay

$X_2 \rightarrow \beta$ – decay

$X_3 \rightarrow \beta$ – decay

$X_4 \rightarrow \alpha$ – decay

8. Fusion of MnO_2 along with KOH and O_2 forms X. Electrolytic oxidation of X yields Y. X undergoes disproportionation reaction in acidic medium to MnO_2 and Y. The Manganese in X and Y is in the form W & Z respectively, then

(a) W & Z are coloured (b) W is diamagnetic and Z is paramagnetic

(c) Both W & Z are tetrahedral in shape (d) Both W & Z involve pπ-dπ bonding for π bond

Solution:

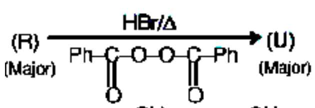
(A, C, D)

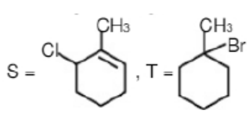
$$MnO_2 \xrightarrow[O_2]{KOH} \underset{(W)}{MnO_4^{2-}} \xrightarrow{e^-} \underset{(Z)}{MnO_4^-}$$

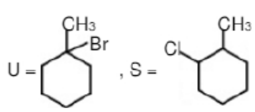
$$\underset{(Z)}{MnO_4^-} \xrightarrow{[H^+]} MnO_2 + MnO_4^-$$

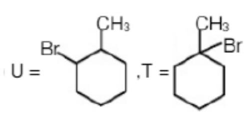
9. $$C_6H_{10}O \xrightarrow[(2)H_2O]{(1)CH_3MgBr} \underset{(Major)}{Q} \xrightarrow{Conc.HCl} \underset{(Major)}{S}$$

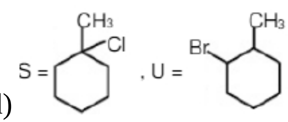
$$\underset{(Major)}{Q} \xrightarrow[360K]{20\%H_3PO_4} \underset{(Major)}{R} \xrightarrow[(2)Br_2/hv]{(1)H_2/Ni} \underset{(Major)}{T}$$



(a) 

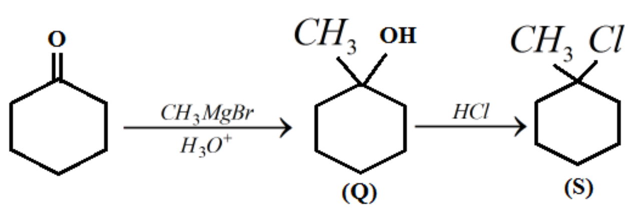
(b) 

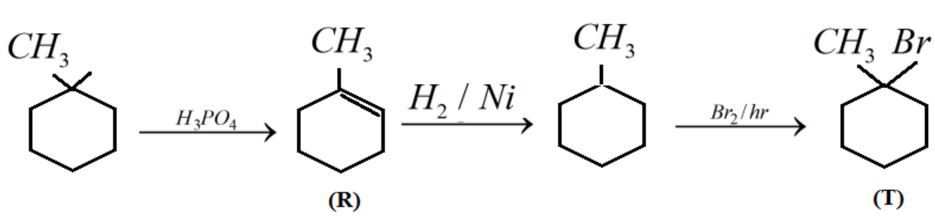
(c) 

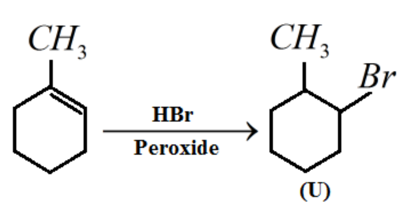
(d) 

Solution:

(C, D)







10. Which of the following are true.

(a) Monosachharides can not be hydrolysed to give polyhydroxy aldehydes and ketones.

(b) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose

(c) Oxidation of glucose with bromine water gives glutamic acid.

(d) The two six membered hemiacetal form of D(+) glucose are anomers.

Solution:

(A, B, D)

11. Identify the option where all four molecules possess permanent dipole moment at room temperature.

(a) BF, O_3, SF_6, XeF_6

(b) $BeCl_2, CO_2, BCl_3, CHCl_3$

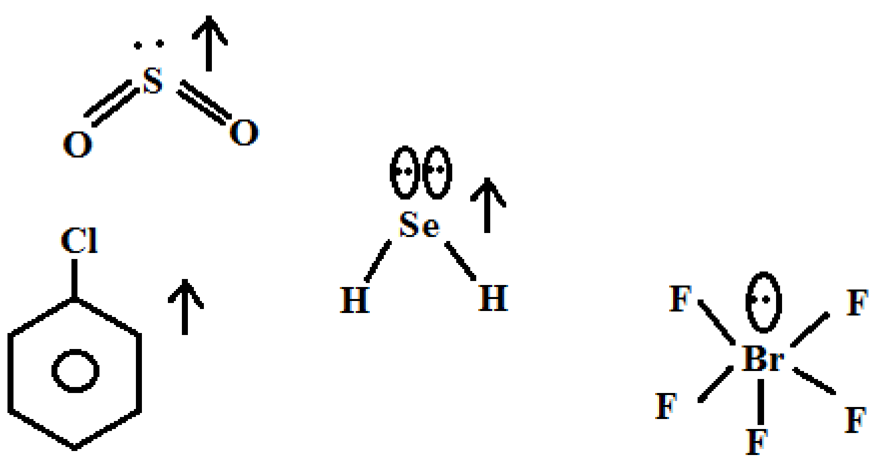
(c) $SO_2, C_6H_5Cl, H_2Se, BrF_5$

(d) $NO_2, NH_3, POCl_3, CH_3Cl$

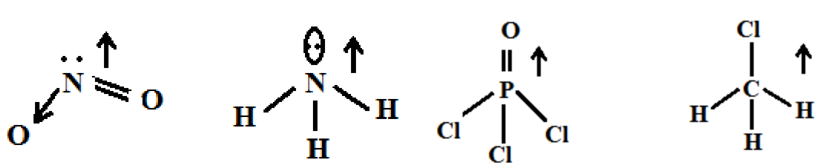
Solution:

(C, D)

(C) $\rightarrow$



(D) $\rightarrow$



12. Which of the following is/are correct regarding root mean square speed (U_{rms}) & average translation K.E. (E_{av}) of molecule in a gas at equilibrium.
    (a) E_{av} is doubled when its temperature is increased 4 times
    (b) U_{rms} is inversely proportional to the square root of its molecular mass
    (c) E_{av} at a given temperature doesn't depend on its molecular mass
    (d) U_{rms} is doubled when its temperature is increased 4 times

Solution:

(B, C, D)

$E_{av} = \frac{3}{2}RT$ (independent of Mass)

$u_{rms} = \sqrt{\frac{3RT}{M}}$

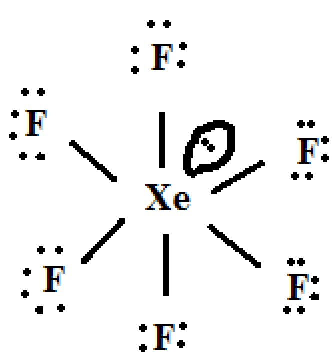
SECTION – 3

13. $XeF_4 + O_2F_2 \longrightarrow$ product. The total number of lone pairs on the xenon containing product is: (l)

Solution:

(19)

$XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$



Distorted octahedral shape

14. For the following reaction, equilibrium constant K_c at 298 K is 1.6×10^{17}

$$Fe^{2+}_{(aq)} + S^{2-}_{(aq)} \rightleftharpoons FeS(s)$$

When equal volume of 0.06 M Fe^{+2} and 0.2 M S^{-2} solution are mixed, then equilibrium concentration of Fe^{+2} is found to be Y × 10^{-17} M. Y is:

Solution:

8.93 or 8.92

$$Fe^{2+} + S^{2-} \longrightarrow FeS$$

ini	0.06	0.2	0
After mix	0.03	0.1	
At 0 q/m	x	0.07	

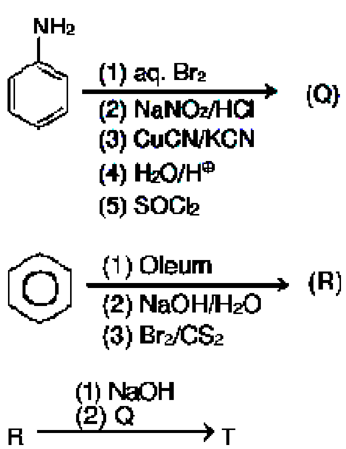
$$K_C = \frac{1}{(x)[0.07]} = 1.6 \times 10^{17}$$

$$\therefore \left[Fe^{2+}\right] = 8.928 \times 10^{-17}$$

$$= y \times 10^{-17}$$

$$\therefore y = 8.93 \; or \; 8.92$$

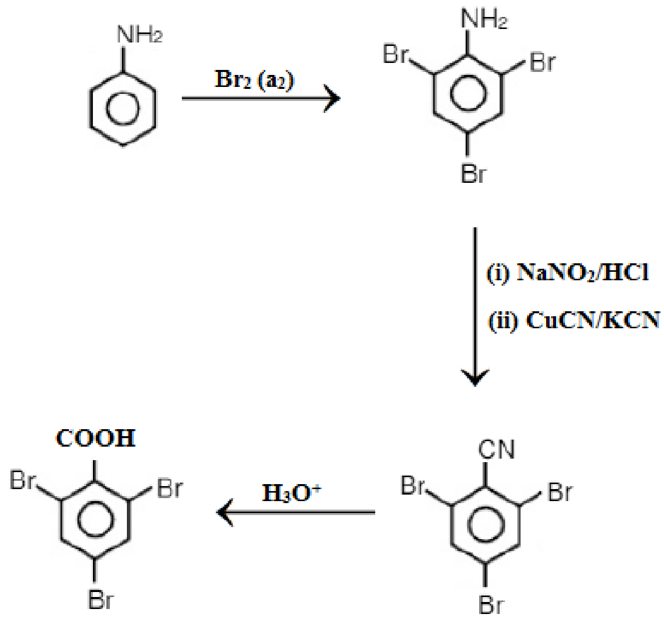
15.

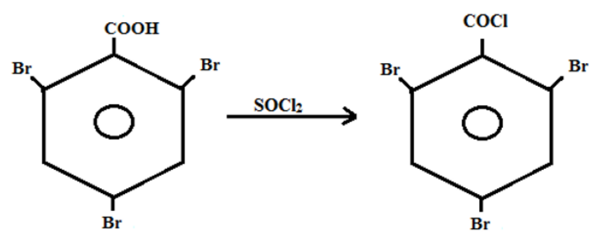


Number of atoms of Br in compound 'T'

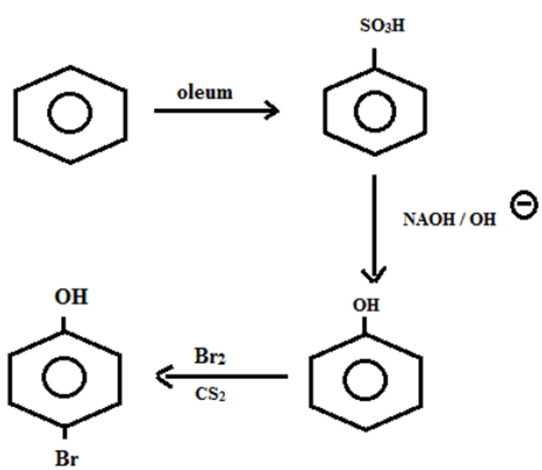
Solution:

(4)

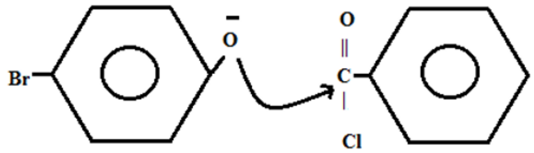

NH2
Br2 (a2)
NH2
Br
Br
Br
(i) NaNO2/HCl
(ii) CuCN/KCN
COOH
Br
Br
Br
H3O+
CN
Br
Br
Br

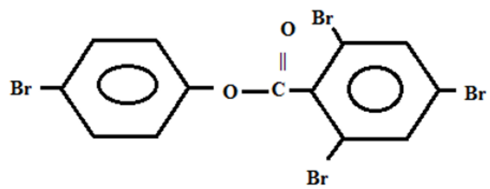

COOH
Br
Br
Br
SOCl2
COCl
Br
Br
Br

(Q)


oleum
SO3H
NAOH / OH
OH
Br2
CS2
OH
Br

(R)





(T)

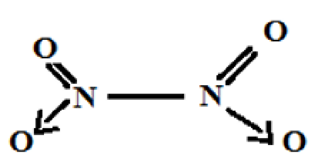
Total Br atoms = 4

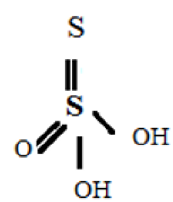
16. Which of the following compounds contain bond between same type of atoms.

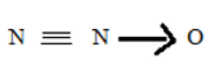
$N_2O_4, B_3N_3H_6, H_2S_2O_3, N_2O, H_2S_2O_8, B_2H_6$

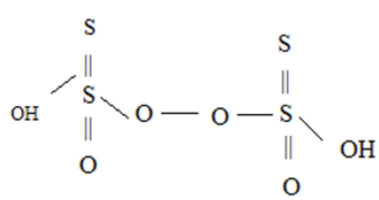
Solution:

$N_2O_4, H_2S_2O_3, N_2O, H_2S_2O_8$









17. A + B + C $\rightarrow$ Product

Ex. No	[A]	[B]	[C]	Rate of reaction
1.	0.2	0.1	0.1	6×10^{-5}
2.	0.2	0.2	0.1	6×10^{-5}
3.	0.2	0.1	0.2	1.2×10^{-4}

4.	0.3	0.1	0.1	9×10^{-5}

When [A] = 0.15

[B] = 0.25

[C] = 0.15

Rate of reaction is Y × 10^{-5} M/s Find Y.

Solution:

(6.75)

Let $r = K[A]^x[B]^y[C]^z$

From (1), (2) $\rightarrow$ y = 0

(1), (3) $\rightarrow$ z = 1

(1), (4) $\rightarrow$ x = 1

$\therefore$ rate law becomes

$$r = K[A]^1[B]^0[C]^1$$

From (2)

$$K = 3\times10^{-3}$$

When $[A]=0.15, [B]=0.25, [C]=0.15$

$$r = 3\times10^{-3}[0.15]^1[0.15]^1$$

$$= 6.75\times10^{-5} mol\, l^{-1}s^{-1}$$

$$\Rightarrow y\times10^{-6}$$

$$\therefore y = 6.75$$

18. On dissolving 0.5 g of non-volatile, non-ionic solute to 39 g of benzene, its vapour pressure decreases from 650 mm of Hg to 640 mm of Hg. The depression of freezing point of benzene (in K) upon addition of the solute is _________.

[Given data: Molar mass & molar freezing point depression of benzene is 78 g mol^{-1} & 5.12 K Kg mol^{-1}]

Solution:

(1.02)

$$\frac{P^\circ - P_S}{P_S} = \frac{n_{solute}}{n_{solvent}}$$

$$\frac{650-640}{640} = \frac{1\times 0.5\times 78}{M\times 39}$$

$$\therefore M = 64g$$

$$\Delta T_f = K_f m = 5.12\times\frac{0.5\times 1000}{64\times 39}$$

$$\therefore \Delta T_f = 1.02$$