DATE: **MATHS**

CLASS: **JEE ADVANCE PAPER 2**

ALL CENTRE

SECTION - 1

1. Let f: R → R be given by f(x) = (x – 1) (x – 2) (x – 5). Define F(x) = $\int_0^x f(t)\,dt, x > 0.$ Then which of the following options is/are correct?

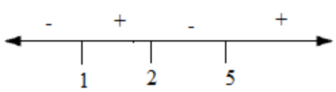
(a) F has a local minimum at x = 1
(b) F has a local maximum at x = 2
(c) F(x) ≠ 0 for all x ϵ (0, 5)
(d) F has two local maxima and one local minimum in $(0, \infty)$

Solution:

f(x) = (x – 1) (x – 2) (x – 5)

Given F(x) = $\int_0^x f(t).dt$

$$F'(x) = (x-1)(x-2)(x-5)$$



At x = 1 and x = 5, $F'(x)$ changes from – to +

∴ F(x) has two local minima points at x = 1 and x = 5

F(x) has one local maxima point at x = 2.

2. For a ϵ R, |a| > 1, let $\lim_{n\to\infty}\left(\dfrac{1+\sqrt[3]{2}+.....\sqrt[3]{n}}{n^{7/3}\left(\dfrac{1}{(an+1)^2}+\dfrac{1}{(an+2)^2}+....+\dfrac{1}{(an+n)^2}\right)}\right) = 54.$ Then the possible value(s) of a is/are:

(a) 8
(b) – 9
(c) – 6
(d) 7

Solution:

$$\lim_{n\to\infty}\frac{\sqrt[3]{1}+\sqrt[3]{2}+......+\sqrt[3]{n}}{n^{7/3}\left[\frac{1}{(an+1)^2}+\frac{1}{(\mathrm{an}+2)^2}+.....+\frac{1}{(\mathrm{an}+\mathrm{n})^2}\right]}=54$$

$$\Rightarrow \lim_{n\to\infty}\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}}{\frac{1}{n}\left[\frac{n^2}{(an+1)^2}+\frac{n^2}{(\mathrm{an}+2)^2}+.....+\frac{n^2}{(\mathrm{an}+\mathrm{n})^2}\right]}=54$$

$$\Rightarrow \frac{\int_0^1 x^{1/3}dx}{\int_0^1 \frac{dx}{(\mathrm{a}+\mathrm{x})^2}}=54 \qquad \Rightarrow \frac{\left[\frac{3}{4}x^{4/3}\right]_0^1}{\left[\frac{-1}{a+x}\right]_0^1}=\frac{3/4}{\frac{1}{a}-\frac{1}{a+1}}=54$$

$$\Rightarrow \frac{(\mathrm{a}+1)-\mathrm{a}}{\mathrm{a}(\mathrm{a}+1)}=\frac{3}{4}\times\frac{1}{54} \qquad \Rightarrow \frac{1}{a(\mathrm{a}+1)}=\frac{1}{72} \qquad \Rightarrow a(\mathrm{a}+1)=72$$

$$\Rightarrow a=8\,or\,a=-9$$

3. Three lines

$$L_1:\vec{r}=\lambda\hat{i},\lambda\in R,$$
$$L_2:\vec{r}=\vec{k}+\mu\hat{j},\mu\in R\,and$$
$$L_3:\vec{r}=\hat{i}+\hat{j}+v\hat{k},v\in R$$

are given. For which point(s) Q and L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear?

(a) $\hat{k}+\hat{j}$ (b) $\hat{k}$ (c) $\hat{k}+\frac{1}{2}\hat{j}$ (d) $\hat{k}-\frac{1}{2}\hat{j}$

Solution:

P (λ, 0, 0), Q (0, μ, 1), R (1, 1, r)

Given $\overrightarrow{PQ}=k.\overrightarrow{PR}\Rightarrow\frac{\lambda}{\lambda-1}=\frac{-\mu}{-1}=\frac{-1}{-r}$

$\therefore$ μ cannot take the values 0 and 1

4. Let F: R → R be a function. We say that f has

PROPERTY 1 $if \lim\limits_{h\to 0}\dfrac{f(\mathrm{h})-\mathrm{f}(0)}{\sqrt{|\mathrm{h}|}}$ exists and is finite and

PROPERTY 2 $f \lim\limits_{h\to 0}\dfrac{f(\mathrm{h})-\mathrm{f}(0)}{h^2}$ exists and is finite

Then which of the following options is/are correct?

(a) f(x) = x|x| has PROPERTY 2

(b) F(x) = $x^{2/3}$ has PROPERTY 1

(c) f(x) = sin x has PROPERTY 2

(d) f(x) = |x| has PROPERTY 1

Solution:

(a) f (x) = x|x|

$$\underset{h\to 0}{Lt}\frac{f(\mathrm{h})-\mathrm{f}(0)}{h^2}=\underset{h\to 0}{Lim}\frac{h|\mathrm{h}|-0}{h^2}$$ which does not exist.

(b) $\underset{h\to 0}{Lim}\dfrac{h^{2/3}-0}{\sqrt{|\mathrm{h}|}}=0$

(c) $\underset{h\to 0}{Lim}\dfrac{\sin\mathrm{h}-0}{h^2}\ does\ not\ exist$

(d) $\underset{h\to 0}{Lim}\dfrac{|\mathrm{h}|-0}{\sqrt{|\mathrm{h}|}}=0$

5. For non-negative integers n, let

$$\mathrm{f(n)}=\frac{\sum\limits_{k=0}^{n}\sin\left(\dfrac{k+1}{x+2}\pi\right)\sin\left(\dfrac{k+2}{n+2}\pi\right)}{\sum\limits_{k=0}^{n}\sin^2\left(\dfrac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}$ x takes value in [0, π], which of the following options is/are correct?

(a) sin (7 $\cos^{-1}$ f(5)) = 0

(b) $f(4)=\dfrac{\sqrt{3}}{2}$

(c) $\lim\limits_{n\to\infty} f(\mathrm{n})=\dfrac{1}{2}$

(d) If α = tan ($\cos^{-1}$ f(6)), then $\alpha^2 + 2\alpha - 1 = 0$

Solution:

$$f(\text{n})=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\cdot\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}2\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

$$=\frac{\sum_{k=0}^{n}\cos\frac{\pi}{n+2}-\cos\left(\frac{2k+3}{n+2}\right)\pi}{\sum_{k=0}^{n}2\sin^2\left(\frac{k+1}{n+2}\right)\pi}$$

$$=\frac{(\text{n}+1)\cos\frac{\pi}{n+2}-\frac{\cos\left(\frac{n+3}{n+2}\right)\pi\cdot\sin\left(\frac{n+1}{n+2}\right)\pi}{\sin\frac{\pi}{n-2}}}{(\text{n}+1)-\frac{\cos\pi\cdot\sin\left(\frac{n+1}{n+2}\right)\pi}{\sin\left(\frac{\pi}{n+2}\right)}}$$

$$=\frac{(\text{n}+1)\cos\left(\frac{\pi}{n+2}\right)+\cos\left(\frac{n+3}{n+2}\right)\pi}{(\text{n}+1)+1}$$

$$=\cos\left(\frac{\pi}{n+2}\right)$$

(A) $\alpha=Tan\left(\cos^{-1}f(6)\right)=Tan\cos^{-1}\left(\cos\pi/8\right)=Tan\,\pi/8$

$$\alpha^2+2\alpha-1=Tan^2\,\pi/8+2Tan\,\pi/8-1$$

$$Tan2\left(\frac{\pi}{8}\right)=\frac{2Tan\,\pi/8}{1-Tan^2\,\pi/8}$$

$$\Rightarrow 1=\frac{2\alpha}{1-\alpha^2}\quad\Rightarrow\alpha^2+2\alpha-1=0$$

$\therefore$ option (A) is correct.

(B) $\lim_{n\to\infty}f(\text{x})=\lim_{n\to\infty}\cos\left(\frac{\pi}{n+2}\right)=\lim_{\frac{1}{n}\to 0}\cos\left(\frac{\pi/n}{1+2/n}\right)=1$

Option (B) correct.

$$(C)\,f(4)=\cos\left(\frac{\pi}{4+2}\right)=\cos{\pi}/{6}=\sqrt{3}/{2}$$

Option (C) wrong

$$(D)\sin\left[7\cos^{-1}f(5)\right]=\sin\left[7\cos^{-1}\left(\cos{\pi}/{7}\right)\right]=\sin\left[7\times\frac{\pi}{7}\right]=0$$

6. Let $P_1=I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, P_2=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}, P_3=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}, P_4=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}, P_5=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix},$

$P_6=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$ and $X=\sum_{k=1}^{6}P_K\begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}P_K^T$

Where P_K^T denotes the transpose of the matrix P_K. Then which of the following options is/are correct?

(a) X – 30I is an invertible matrix

(b) The sum of diagonal entries of X is 18

(c) $If\ X\begin{bmatrix}1\\1\\1\end{bmatrix}=\alpha\begin{bmatrix}1\\1\\1\end{bmatrix}, then\, \alpha=30$

(d) X is a symmetric matrix

<u>Solution:</u>

From the given data it is clear that

$$P_1=P_1^T=P_1^{-1}$$
$$P_2=P_2^T=P_2^{-1}$$

$$P_6=P_6^T=P_6^{-1}$$

And Let $A=\begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}$

Here $A^T = A$ → A is symmetric matrix

$$X^T=\left(P_1AP_1^T+......+P_6AP_6^T\right)^T$$

$= P_1 A^T P_1^T + ... + P_6 A^T P_6^T$

= X

$\therefore X \text{ is symmetric}$

$$Let\ B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$XB = P_1 A P_1^T G + P_2 A P_2^T B + ... + P_6 A P_6^T B$$

$$= P_1 AB + P_2 AB + ... + P_6 AB$$

$$= (P_1 + P_2 + P_3 + ... + P_6)\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} = 30B \qquad \Rightarrow \propto = 30$$

Since $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 30\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow (X - 30I)$ B = 0 has a nontrivial solution $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow (X - 30I) = 0$

$X = P_1 A P_1^T + ... + P_6 A P_6^T$

$$\text{Trace}\left(X\right) = tr\left(P_1AP_1^T\right) + ... + Tr\left(P_6AP_6^T\right)$$

= (2 + 0 + 1) + …. + (2 + 0 + 1) = 3 + 3 + …. (6 times) = 18

7. Let x ϵ R and let $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \textit{and } R = PQP^{-1}$

Then which of the following options is/are correct?

(a) For x = 1, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which R $\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(b) There exists a real number x such that PQ = QP

(c) det R = det $\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8, \textit{ for all } x \,\varepsilon\, R$

(d) for x = 0, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}, \textit{then } a + b = 5$

Solution:

R = PQP^{-1}

$|R| = |P||Q|.|P^{-1}|$

$\Rightarrow \det Q = 2(24) - x(0) + x(-4x) = 48 - 4x^2$

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} . Q(X = 0) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

R = PQR^{-1}

$$=\begin{bmatrix}2&4&6\\0&8&12\\0&0&18\end{bmatrix}.\frac{1}{6}\begin{bmatrix}6&-3&0\\0&3&-2\\0&0&2\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}12&6&4\\0&24&8\\0&0&36\end{bmatrix}=\begin{bmatrix}2&1&2/3\\0&4&4/3\\0&0&6\end{bmatrix}$$

$$\left(R-6I\right)\begin{pmatrix}1\\a\\b\end{pmatrix}=\begin{pmatrix}-4&1&2/3\\0&-2&4/3\\0&0&0\end{pmatrix}\begin{pmatrix}1\\a\\b\end{pmatrix}=\begin{bmatrix}-4&+a&+\frac{2b}{3}\\0&-2a&+4b/3\\0&0&0\end{bmatrix}$$

$$-4+a+\frac{2b}{3}=0 \text{ and } -2a+\frac{4b}{3}=0\Rightarrow a=2\ \&\ b=3$$

$$\therefore \quad a+b=5$$

$PQ = QP\Rightarrow x+4+x=2+2x+0\Rightarrow$ No value exist

8. Let $f(\mathrm{x})=\dfrac{\sin\pi x}{x^2}, x>0$

Let $x_1 < x_2 < x_3 < \ldots. < x_n < \ldots$ be all the points of local maximum of f
and $y_1 < y_2 < y_3 < \ldots. < y_n < \ldots.$. be all the points of local minimum of f.
Then which of the following options is/are correct?

(a) $|x_n - y_n| > 1$ for every n

(b) $x_1 < y_1$

(c) $x_n \in \left(2n, 2n+\frac{1}{2}\right)$ *for every n*

(d) $x_{n+1}-x_n > 2$ *for every n*

Solution:

$$f(\mathrm{x})=\frac{\sin\pi x}{x^2} \quad\Rightarrow\quad f'(\mathrm{x})=\frac{x^2\cdot(\cos\pi\,\mathrm{x})\cdot(\pi)-\sin\pi\,\mathrm{x}\cdot(2\,\mathrm{x})}{x^4}$$

$$\Rightarrow\quad f'(\mathrm{x})=\frac{2x\cos\pi x\left(\frac{\pi x}{2}-\tan\pi x\right)}{x^4}$$

By using graph we can say that option (1) (3) (4) are correct.

SECTION – 2

1. The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals

Solution:

$$\sec^{-1}\pi\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$

$$=\sec^{-1}\left(\frac{-1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\operatorname{cosec}\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right)$$

$$=\sec^{-1}\left(\frac{-1}{4}\sum_{k=0}^{10}\frac{2}{\sin\left(\frac{7\pi}{6}+k\pi\right)}\right)$$

$$=\sec^{-1}\left(\frac{-1}{2}\sum_{k=0}^{10}\frac{1}{(-1)^{k+1}\sin\frac{\pi}{6}}\right)$$

$$=\sec^{-1}\left(-\sum_{k=0}^{10}\frac{1}{(-1)^{k+1}}\right)=\sec^{-1}(1)=0$$

2. Let |X| denote the number of elements in set X. Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \le |B| < |A|$, equals.

Solution:

The number of ordered pairs of (A, B) are

$6c_1\,(6c_2 + 6c_3 + \ldots + 6c_6) + 6c_2\,(6c_2(6c_3 + 6c_4 \ldots + 6c_6) + 6c_3(6c_4 + 6c_5 + 6c_6) + 6c_4(6c_5 + 6c_6) + 6c_5 \,.\, 6c_6$

$= (6c_1 .\, 6c_2 + 6c_1 \,.\, 6c_3 + \ldots + 6c_1 6c_6) + (6c_2.6c_3 + 6c_2.6c_4 + \ldots + 6c_2.6c_6) + (6c_3.6c_4 + 6c_3.6c_5 + 6c_3.6c_6)$

$+ 6c_4.6c_5 + 6c_4.6c_6 + 6c_5.6c_6.$

$= (12c_5 – 6c_1) + (12c_4 – 6c_2) + (12c_3 – 6c_3) + (12c_2 – 6c_4) + (12c_1 – 6c_5)$

$= (12c_1 + 12c_2 + 12c_3 + 12c_4 + 12c_5) – (6c_1 + 6c_2 + \ldots + 6c_5)$

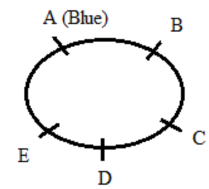
$= 1585 – 62 = 1523.$

3. Five person A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is

Solution:

Maximum number of hats used of same colour are 2.

They cannot be 3 otherwise atleast 2 hats of same colour are consecutive.



Now the hats used are consider as B B G G B

Which can be selected in 3 ways.

It can be R G G B B or R R G B B

The number of ways of distributing blue hat (single one) in 5 persons equal to 5

Now either position B and D are filled by green hats and C and E are filled by Red hats or B & D are filled by

Red hats and C & E are filled by Green hats.

➔2 ways are possible.

Hence number of ways = 3×5×2 = 30 ways.

4. Suppose

$$\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$$, holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals

Solution:

$$\begin{vmatrix} \sum\limits_{k=0}^{n} k & \sum\limits_{k=0}^{n} {}^{n}C_k \cdot k^2 \\ \sum\limits_{k=0}^{n} \cdot {}^{n}C_k \cdot k & \sum\limits_{k=0}^{n} \cdot {}^{n}C_k \cdot 3^k \end{vmatrix} = 0$$

$$\begin{vmatrix} \frac{n(\mathrm{n}+1)}{2} & n \cdot 2^{n-1} + n(\mathrm{n}-1) \cdot 2^{\mathrm{n}-2} \\ n \cdot 2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\Rightarrow \frac{n(\mathrm{n}+1)}{2} \cdot 4^n - n \cdot 2^{2n-1}\left(n \cdot 2^{n-1} + n(\mathrm{n}-1) \cdot 2^{\mathrm{n}-2}\right) = 0$$

$$\Rightarrow \frac{n(\mathrm{n}+1)}{2} \cdot 4^n - n^2 \cdot 2^{2n-2} \cdot - n^2(\mathrm{n}-1) \cdot 2^{2\mathrm{n}-3} \cdot = 0$$

$$\Rightarrow \frac{n(\mathrm{n}+1)}{2} - \frac{n^2}{4} - \frac{n^2(\mathrm{n}-1)}{8} = 0 \quad \Rightarrow \frac{n}{2}\left[n+1-\frac{n}{2}-\frac{n(\mathrm{n}-1)}{4}\right] = 0$$

$$\Rightarrow n = 0\, or\, 4(\mathrm{n}+1) - 2\,\mathrm{n} - 1(\mathrm{n}-1) = 0 \qquad \Rightarrow n = 0\, or\, n = 4$$

$$\sum_{\pi=0}^{4} \frac{4c\pi}{r+1} = \sum_{r=0}^{4} \frac{5cr+1}{5} = \frac{2^5-1}{5} = \frac{31}{5} = 6.20$$

5. The value of the integral $\int\limits_{0}^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$ equals

Solution:

$$I = \int\limits_{0}^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)^5} \cdot d\theta$$

$$I = 3\int\limits_{0}^{\pi/2} \frac{\sqrt{\cos\theta}}{\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)^5} \qquad \rightarrow 1$$

$$I = 3\int_0^{\pi/2} \frac{\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} \qquad \to 2 \qquad \left[\because \int_0^a f(\mathrm{x})\,\mathrm{dx} = \int_0^a f(\mathrm{a}-\mathrm{x})\cdot \mathrm{dx}\right]$$

$$2I = 3\int_0^{\pi/2} \frac{\sqrt{\cos\theta}\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}\cdot d\theta = 3\int_0^{\pi/2} \frac{d\theta}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^4}$$

$$\frac{2I}{3} = \int_0^{\pi/2} \frac{\sec 2\theta \cdot d\theta}{\left(\sqrt{\tan\theta}+1\right)^4}$$

Let $Tan\theta = t^2 \quad \Rightarrow \quad \sec 2\theta \cdot d\theta = 2t\,dt$

$$\frac{2I}{3} = \int_0^{\infty} \frac{2tdt}{(\mathrm{t}+1)^4}$$

$$\frac{I}{3} = \int_0^{\infty}\left[\frac{1}{(\mathrm{t}+1)^3} - \frac{1}{(\mathrm{t}+1)^4}\right]dt$$

$$I = \left[\frac{-3}{2(\mathrm{t}+1)^2} + \frac{1}{(\mathrm{t}+1)^3}\right]_0^{\infty}$$

$$= \frac{3}{2} - 1 = \frac{1}{2}$$

6. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ *and* $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \alpha, \beta\, \varepsilon$. If the projection of $\vec{c}$ on the vector $\left(\vec{a}+\vec{b}\right) is\, 3\sqrt{2}$, then the minimum value of $\left(\vec{c} - \left(\vec{a}\times\vec{b}\right)\right).\vec{c}$ equals

Solution:

$$\vec{a} = 2i + j - k$$

$$\vec{b} = i + 2j + k$$

$$\vec{c} = \alpha\vec{a} + \beta\vec{b} = \alpha(2\mathrm{i} + \mathrm{j} - \mathrm{k}) + \beta(\mathrm{i} + 2\,\mathrm{j} + \mathrm{k})$$

$$= (2\alpha + \beta)\mathrm{i} + (\alpha + 2\beta)\,\mathrm{j} + (\beta - \alpha)\,\mathrm{k}$$

$$\textit{Given}\quad \frac{\vec{c}\cdot(\mathrm{a}+\mathrm{b})}{|\vec{\mathrm{a}}+\vec{\mathrm{b}}|} = 3\sqrt{2}$$

$$\Rightarrow 9(\alpha + \beta) = 18 \quad \Rightarrow \alpha + \beta = 2$$

$$(\vec{\mathrm{c}} - \mathrm{a}\times\mathrm{b})\,\mathrm{c} = \left(\alpha\vec{a} + \beta\vec{b} + \vec{a}\times\vec{b}\right)\cdot\left(\alpha\vec{a} + \vec{b}\beta\right)$$

$$= 6\alpha^2 + 6\alpha\beta + 6\beta^2 = 6\left[\alpha^2 + \alpha(2-\alpha) + (2-\alpha)^2\right]$$

$$= 6\left(\alpha^2 - 2\alpha + 4\right)$$

Minimum value = 18

SECTION – 3

1. Answer the following by appropriately matching the lists based on the information given in the paragraph
Let f(x) = sin(π cosx) and g(x) = cos(2π sinx) be two functions defined for x > 0. Define the following sets whose element are written in the increasing order:

$$X = \{x : f(\mathrm{x}) = 0\}\ ,\quad Y = \{x : f'(\mathrm{x}) = 0\}$$

$$Z = \{x : g(\mathrm{x}) = 0\}\ ,\quad W = \{x : g'(\mathrm{x}) = 0\}$$

List –I contains the sets X,Y,Z and W. List – II contains some information regarding these sets.

List I	**List – II**
(I) X	(P) $\supseteq\left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) Not an arithmetic progression
(IV) W	(S) $\supseteq\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$

(U) $\supseteq\left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$

Which of the following is the only correct combination?

(a) (II), (R), (S) (b) (I), (P), (R) (c) (II), (Q), (T) (d) (I), (Q), (U)

2. Answer the following by appropriately matching the lists based on the information given in the paragraph
Let f(x) = sin(π cosx) and g(x) = cos(2π sinx) be two functions defined for x > 0. Define the following sets whose element are written in the increasing order:

$$X=\{x: f(\mathrm{x})=0\},\quad Y=\{x: f'(\mathrm{x})=0\}$$

$$Z=\{x: g(\mathrm{x})=0\},\quad W=\{x: g'(\mathrm{x})=0\}$$

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	(T) $\supseteq\left\{\frac{\pi}{3},\frac{2\pi}{3},\pi\right\}$
	(U) $\supseteq\left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$

Which of the following is the only correct combination?

(a) (IV), (Q), (T) (b) (IV), (P), (R), (S) (c) (III), (R), (U) (d) (III), (P), (Q), (U)

Solution:

$$f(\mathrm{x})=0\Rightarrow\sin\left(\pi\cos x\right)=0$$

$$\Rightarrow\pi\cos x=n\pi\Rightarrow\cos x=n\Rightarrow\cos x=-1,0,1$$

$$x=\left\{\mathrm{n}\,\pi,(2\,\mathrm{n}\,\pi)\,\pi\!\!\diagup\!_2\right\}$$

$$x=\left\{\frac{n\pi}{2},\mathrm{n}\in\mathrm{I}\right\}$$

$$f'(x) = 0 \Rightarrow \cos(\pi \cos x)(-\pi \sin x) = 0$$

$$\Rightarrow \pi \cos x = (2n+1)\pi/2 \; or \; x = n\pi$$

$$\Rightarrow \cos x = n + \frac{1}{2} \; or \; x = n\pi$$

$$\Rightarrow \cos x = \pm\frac{1}{2} \; or \; x = n\pi$$

$$\therefore y = \left\{2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3}, n\pi\right\}$$

$$g(x) = 0 \Rightarrow \cos(2\pi \sin x) = 0$$

$$\Rightarrow 2\pi \sin x = (2n+1)\pi/2$$

$$\Rightarrow \sin x = \frac{2n+1}{4} = \pm\frac{1}{4}, \pm\frac{3}{4}$$

$$z = \left\{n\pi \pm \sin^{-1}\frac{1}{4}, n\pi \pm \sin^{-1}\frac{3}{4}, n \in I\right\}$$

$$g'(x) = 0 \Rightarrow -\sin(2\pi \sin x)(2\pi \cos x) = 0$$

$$\Rightarrow 2\pi \sin x = n\pi \; or \; x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \sin x = \frac{n}{2} = 0, \pm\frac{1}{2}, \pm 1 \; or \; x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow W = \left\{n\pi, (2n+1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I\right\}$$

(1) Option – 3 (2) Option – 2

3. Answer the following by appropriately matching the lists based on the information given in the paragraph

DATE:	MATHS
CLASS:	JEE ADVANCE PAPER 2
ALL CENTRE	

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions:

(i) centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x_2 = 8\alpha y$.

There are some expression given in the List – I whose values are given in List – II below:

List – I	**List – II**
(I) $2h + k$	(P) 6
(II) $\dfrac{Length\,of\;ZW}{Length\,of\;XY}$	(Q) $\sqrt{6}$
(III) $\dfrac{Area\,of\;triangle\,MZN}{Area\,of\;triangle\,ZMW}$	(R) $\dfrac{5}{4}$
(IV) α	(S) $\dfrac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\dfrac{10}{3}$

Which of the following is the only INCORRECT combination?

(a) (IV), (S) (b) (IV), (U) (c) (III), (R) (d) (I), (P)

Solution:

(ii) Equation of line zw

$C_1 = C_2$

$$\Rightarrow 3x + 4y = 9$$

$$\Rightarrow Dis\tan ce\, of zw\;from\,(0,0)$$

$$\left|\frac{-9}{\sqrt{3^2+4^2}}\right| = \frac{9}{5}$$

Length of xy = $2\sqrt{9-\left(\frac{9}{5}\right)^2} = \frac{24}{5}$

Distance of zw from c

$$\frac{\left|\frac{3\times 9}{5}+4\times\frac{12}{5}-9\right|}{\sqrt{3^2+4^2}} = \frac{6}{5}$$

Length of zw = $2\sqrt{6^2-\frac{6^2}{5^2}} = \frac{24\sqrt{6}}{5}$

$$\frac{length\,of\ zw}{length\,of\ xy} = \sqrt{6}$$

(iii) Area of $\Delta mzN = \frac{1}{2}\cdot Nm\cdot\left(\frac{1}{2}zw\right) = \frac{72\sqrt{6}}{5}$

Area of $\Delta zmw = \frac{1}{2}\cdot zw\cdot(om+op) = \frac{1}{2}.\frac{24\sqrt{6}}{5}.\left(3+\frac{9}{5}\right) = \frac{288\sqrt{6}}{25}$

$$\therefore \frac{\text{Area of }\Delta mzN}{\text{Area of }\Delta zmw} = \frac{5}{4}$$

(iv) Slope of tangent to C_1 at $m = \frac{-1}{4/3} = -\frac{3}{4}$

Equation of Tangent $y = mx - 2\sqrt{1+m^2}$

$$y = \frac{-3x}{4} - 3\sqrt{1+\frac{9}{16}}$$

$$y = \frac{-3x}{4}\frac{-15}{4}$$

$$\Rightarrow x = \frac{-4y}{3} - 5 \qquad \rightarrow 1$$

Tangent to $x^2 = 4(2d)y$ is $x = m'y + \frac{2d}{m^1}$ $\rightarrow 2$

Compare 1 and 2

$$m' = \frac{-4}{3} \; and \; \frac{2\propto}{m^1} = -5 \qquad \Rightarrow \propto = \frac{10}{3}$$

4. Answer the following by appropriately matching the lists based on the information given in the paragraph
Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions:
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(ii) C_1 and C_2 both lie inside C_3, and
(iii) C_3 touches C_1 at M and C_2 at N
Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x_2 = 8\alpha y$.
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(III) $\frac{Area\,of\;triangle\,MZN}{Area\,of\;triangle\,ZMW}$	(R) $\frac{5}{4}$
(IV) α	(S) $\frac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\frac{10}{3}$

Which of the following is the only INCORRECT combination?
(a) (II), (T) (b) (I), (S) (c) (I), (U) (d) (II), (Q)

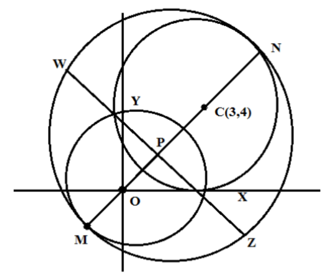
Solution:

$$2r = MN = 3 + \sqrt{3^2 + 4^2} + 4 = 12$$

$$\Rightarrow r = 6$$

Centre c of circle c_3 lies on $y = \frac{4}{3}x$

Let $c\left(h, \frac{4}{3}h\right)$



$$OC = MC - OM = \frac{12}{2} - 3 = 3$$

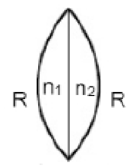
$$\sqrt{h^2 + \frac{16}{9}h^2} = 3 \Rightarrow h = \frac{9}{5}$$

$$k = \frac{4}{3}h = \frac{12}{5} \Rightarrow 2h + k = 6$$

PHYSICS – JEE ADVANCED PAPER – 2

SECTION – 1

1. Consider two plane convex lens of same radius of curvature and refractive index n_1 and n_2 respectively. Now consider two cases:



Case – I: When $n_1 = n_2 = n$, then equivalent focal length of lens is f_0

Case – II: When $n_1 = n$, $n_2 = n + \Delta n$, then equivalent focal length of lens is $f = f_0 + \Delta f_0$

Then correct options are:

(a) If $\Delta n/n > 0$, then $\Delta f_0/f_0 < 0$

(b) $|\Delta f_0/f_0| < |\Delta n/n|$

(c) If $n = 1.5$, $\Delta n = 10^{-3}$ and $f_0 = 20$ cm then $|\Delta f_0| = 0.02$ cm

(d)

Solution:

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{f}\right) \Rightarrow \frac{1}{f_0} = \frac{2(n-1)}{R} \qquad \ldots (1)$$

$$\frac{1}{f_2} = (n+\Delta n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f+\Delta f_0} = \left(\frac{n-1}{R}\right) + (n+\Delta n-1)\left(\frac{1}{f}\right)$$

$$= \frac{2n+\Delta n-2}{R} \qquad \ldots (2)$$

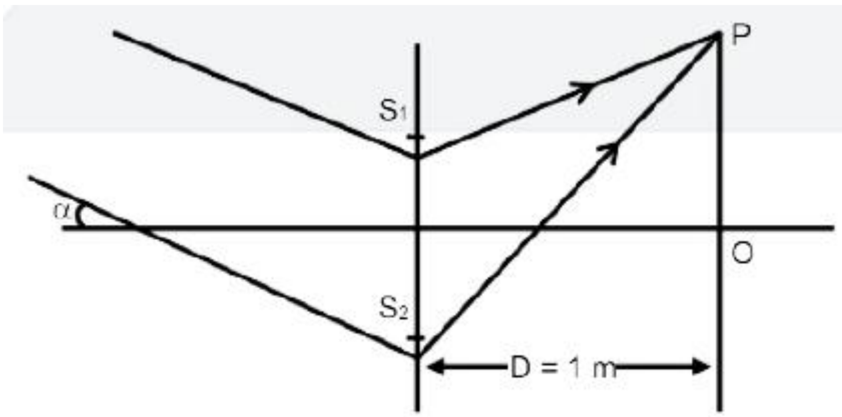
$$\left(\frac{f_0+\Delta f_0}{f_0}\right) = \frac{(2n-1)/R}{(2n+\Delta n-2)/R}$$

$$\frac{1+\Delta f_0}{f_0} = \frac{2(n-1)}{2n+\Delta n-2}$$

$$\Delta f_0 = -2\times 10^{-2}$$

A, C

2. In YDSE monochromatic light of wavelength 600 nm incident of slits as shown in figure.



If S_1S_2 = 3mm, OP = 11 mm then

(a) If $\alpha = \frac{0.36}{\pi}$ degree then destructive interfaces at point P.

(b) If $\alpha = \frac{0.36}{\pi}$ degree then constructive interfaces at point O.

(c) If α = 0 then constructive interfaces at O

(d) Fringe width depends an α

Solution:

d = 3mm OP = 11 mm

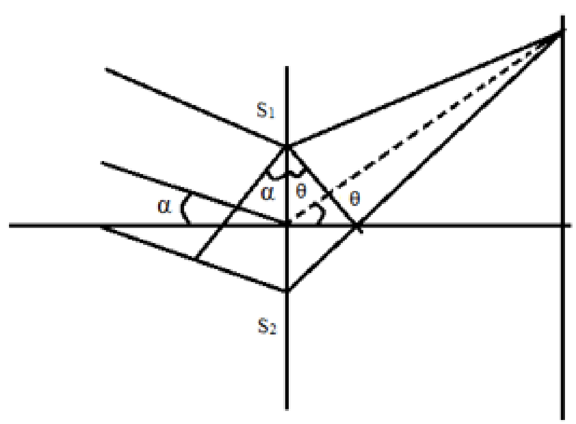
$$\Delta x = d\sin\alpha + d\sin\theta$$

$$= d\alpha + \frac{dy}{D}$$

(A) $\Delta x = 3\times10^{-3}\times\frac{.36}{\pi}\times\frac{\pi}{180}+\frac{3\times11\times10^{-6}}{1} = 3900$

$$3900 = (2n-1)\frac{\lambda}{2} \Rightarrow n = 7$$

Dest



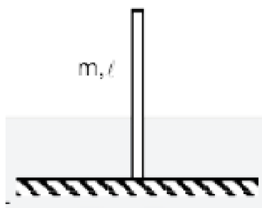
(B) $\Delta x = 3mm\times\frac{.36}{\pi}\times\frac{\pi}{180} = 600\,nm$

$$600nm = n600nm$$

$\Rightarrow n = 1$ const

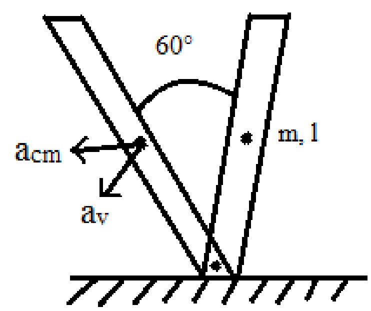
(C) $\alpha = 0$ $\Delta x = 0$ $\therefore$ const

3. A uniform rigid rod of mass m & length l is released from vertical position on rough surface with sufficient friction for lower end not to slip as shown in figure. When rod makes angle 60° with vertical then find correct alternative/s



(a) $\alpha = \frac{2g}{\ell}$ (b) $\omega = \sqrt{\frac{3g}{2\ell}}$ (c) $N = \frac{mg}{16}$ (d) $a_{radial} = \frac{3g}{4}$

Solution:



$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}T_0\omega^2 = -\Delta U$$

$$\frac{1}{2}\frac{ml^2}{3}\omega^2 = -\left(-mg\frac{L}{4}\right)$$

$$\omega = \sqrt{\frac{3g}{u}}$$

$$a_{radial} = \frac{\omega^2\ell}{2} = \frac{3g}{u}\times\frac{\ell}{2} = \frac{3g}{4}$$

$$\tau = I_0\alpha$$

$$\alpha = \frac{mg\frac{l}{2}\sin 60}{l^2} = \frac{3\sqrt{3}g}{4l}$$

$$a_v = \left(\alpha \frac{l}{2}\right)\sin 60° + \omega^2 \frac{l}{2}\cos 60°$$

$$= \frac{3\sqrt{3}g}{8}\frac{\sqrt{3}}{2} + \frac{3g}{8}$$

$$mg - N = ma_v$$

$$N = \frac{mg}{16}$$

4. Monoatomic gas A having 5 mole is mixed with diatomic gas B having 1 mole in container of volume V_0. Now the volume of mixture is compressed to $\frac{V_0}{4}$ by adiabatic process. Initial pressure and temperature of gas mixture is P_0 and T_0. [given $2^{3.2} = 9.2$]
   Choose correct option:

   (a) $\gamma_{mix} = 1.6$

   (b) Final pressure is between $9P_0$ and $10P_0$

   (c) $|W.D| = 13RT_0$

   (d) Average Translational kinetic energy

Solution:

$$V_{mix} = \frac{n_1 C_{P_1} + n_2 l_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{8}{5}$$

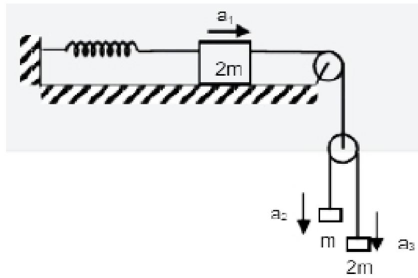
$$W = \frac{P_1V_1 - P_2V_2}{V - 1}$$

$$P_0 V_0^{815} = P_2\left(\frac{V_0}{9}\right)^{8/5}$$

$$P_2 = 9.2P_0$$

$$\omega = \frac{\left(P_0V_0 - 92P_0\frac{V_0}{4}\right)}{3/5} = -13RT_0$$

5. The given arrangement is released from rest when spring is in natural length. Maximum extension in spring during the motion is x_0. a_1, a_2 and a_3 are accelerations of the blocks. Make the correct options



(a) $a_2 - a_1 = a_1 - a_3$

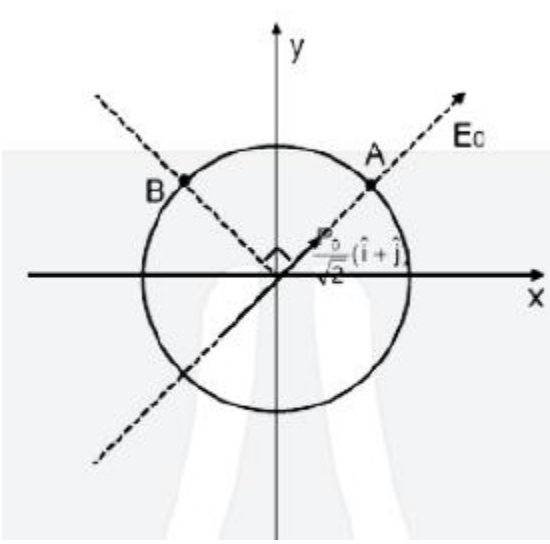
(b) $x_0 = \frac{4mg}{3k}$

(c) Velocity of 2m connected to spring when elongation is $\frac{x_0}{2}$ is $v = \frac{x_0}{2}\sqrt{\frac{3k}{14m}}$

(d) Acceleration a_1 at $\frac{x_0}{4}$ is $\frac{3kx_0}{42m}$

Solution:

6. A dipole of Dipole moment $p = \frac{p_0}{\sqrt{2}}\left(\hat{i} + \hat{j}\right).$ is placed at origin. Now a uniform external electric field at magnitude E_0 is applied along direction of dipole. Two points A and B are lying on a equipotential surface of radius R centered at origin. A is along axial position of dipole and B is along equatorial position. There correct option are:



(a) Net electric field at point A is $3E_0$

(b) Net electric field at point B is Zero

(c) Radius of equatorial surface $R = \left(\frac{kp_0}{E_0}\right)^{1/3}$ (d) Radius of equatorial surface $R = \left(\frac{\sqrt{2}kp_0}{E_0}\right)^{1/3}$

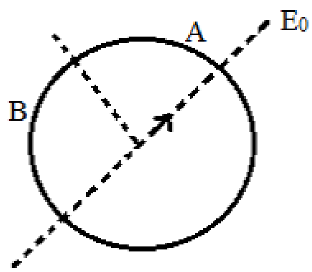
Solution:

$$P = \frac{P_0}{\sqrt{2}}(x+1)$$

$$\frac{KP_0}{r^3} = E_0$$

$$(E_A)_{net} = \frac{2KP_0}{r^3} + E_0 = 3E_0$$

$$(E_B)_{net} = 0$$



7. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from state n = 1 to n = 4. Immediately after electron jumps to n = m state by emitting a photon of wavelength λ_e. Let change in momentum of atom due to the absorption and the emission are ΔP_a and Δp_e respectively. If $\lambda_a / \lambda_e = 1/5$. Which of the following is correct

(a) m = 2

(b) $\Delta P_a/P_e = 1/2$

(c) $\lambda_e = 418$ nm

(d) Ratio of K.E. of electron in the state n = m to n = 1 is ¼.

Solution:

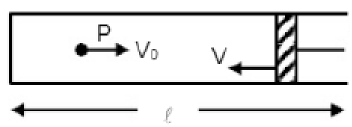
$$\frac{\lambda_a}{\lambda_c} = \frac{E_4 - E_1}{E_4 - E_m} = \frac{\left(1 - \frac{1}{16}\right)}{\left(\frac{1}{m^2} - \frac{1}{16}\right)} = \frac{1}{5}$$

$$\Rightarrow m = 2$$

$$\lambda_c = \frac{12400 \times 4}{13.6} = 3647$$

$$\frac{K_2}{K_1}=\frac{1^2}{2^2}=\frac{1}{4}$$

8. In a cylinder a heavy piston is moving with speed v as shown diagram and gas is filled inside it. A gas molecule is moving with speed v_0 towards moving piston. Then which of the following is correct (Assume v <<<< $v_0 \frac{\Delta \ell}{\ell}$) and collision is elastic)



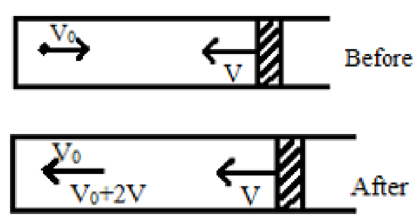
(a) change in speed after collision is 2V

(b) change is speed after collision is $2v_0\frac{\Delta \ell}{\ell}$

(c) rate of collision is $\frac{V}{\ell}$

(d) When piston is at $\frac{\ell}{2}$ its kinetic energy will be four times

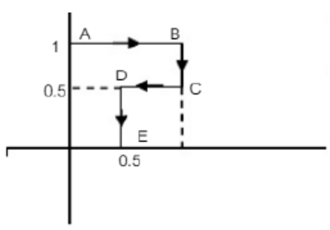
Solution:



Change in speed is $(2V+V_0-V_0)=2V$

SECTION – 2

9. If $f=\alpha y\hat{i}+2\alpha x\hat{j}$ calculate the work done if a particle moves along path as shown in diagram.



Solution:

$$d\omega=\alpha y dx+2\alpha x dx$$

$$\omega_{A\to B}=\int \alpha y\,dx=\alpha 1\int_0^1 dx=\alpha$$

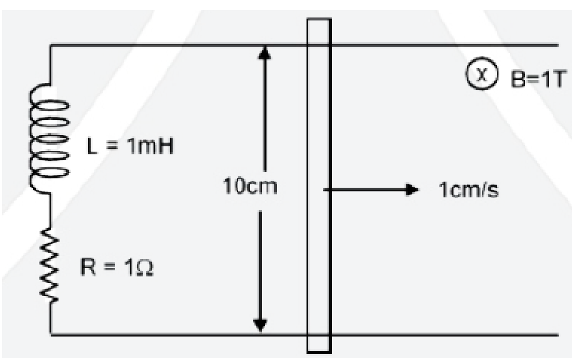
$$\omega_{B\to C} = 2\alpha 1\int_{1}^{0.5} dy = -\alpha$$

$$\omega_{C\to D} = \int_{1}^{0.5} \alpha y\, dx = -\frac{\alpha}{4}$$

$$\omega_{D\to E} = 2\times\alpha\int_{0.5}^{0} x\, dy = -\frac{\alpha}{2}$$

$$\omega = -3/4$$

10. In a given circuit inductor of L = 1mH and resistance R = 1Ω are connected in series to ends of two parallel conducting rods as shown. Now a rod of length 10 cm is moved with constant velocity of 1 cm/s in magnetic field B = 1T. If rod starts moving at t = 0 then current in circuit after 1 millisecond is $x\times10^{-3}A.$ Then value of x is: (given $e^{-1} = 0.37$)



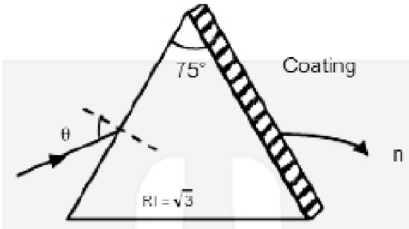
Solution:

$$e = (V\times B)dl = 10^{-3} vdf$$

$$i = 10^{-3}\left(1-e^{-1}\right)$$

$$i = 0.63 mA$$

11. A prism is shown in the figure with prism angle 75° and refractive index $\sqrt{3}.$ A light ray incidents on a surface at incident angle θ. Other face is coated with a medium of refractive index n. For $\theta \le 60°$ ray suffers total internal reflection find value of n^2.



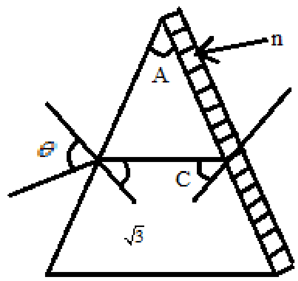
Solution:

$$\sin\theta = \frac{n}{\sqrt{3}}$$

$$\sin\theta = \sqrt{3}\sin(75 - C)$$

$$@\theta = 60 \ \underline{T2R}$$

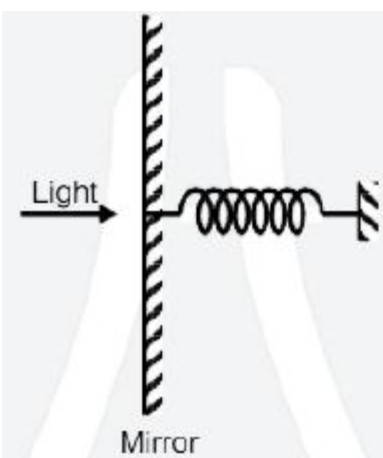
$$\sin 60 = \sqrt{3}\sin(75 - C)$$



C = (45°)

$$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \quad n = \frac{\sqrt{3}}{\sqrt{2}} \quad n^2 = 1.5$$

12. Perfectly reflecting mirror of mass M mounted on a spring constitute a spring mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} m^{-2}$ where h is plank constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strikes the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm. If the value of N is $x \times 10^{12}$, then find the value of x.



Solution:

Photons are reflected

$\therefore \quad MV = \frac{2Nh}{\lambda}$ mean

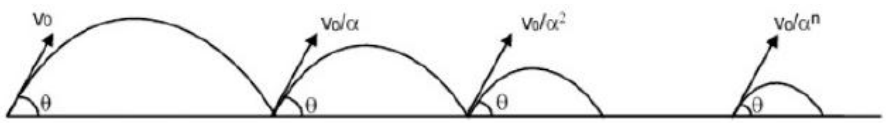
$V_{mean} = \omega A$ A = 1 min

$N = \frac{m\omega\left(10^{-6}\right)\lambda}{2h}$

$N = \frac{4\pi M\omega}{h} \times 10^{-12}$

$\therefore X = 1$

13. A particle is projected with speed v_0 at an angle $\theta\ (\theta \neq 90°)$ with horizontal and it bounce at same angle with horizontal. If average velocity of journey is 0.8 v_0 where v_0 is average velocity of first projectile then α is.

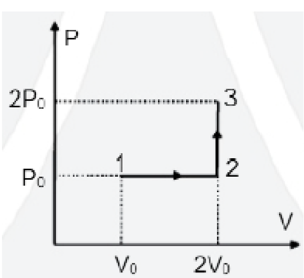


14.

Match the column

A sample of monoatomic gas undergoes different thermodynamic process. Q = Heat given to the gas, W = Work done by the gas, U = Change in internal energy of the gas.

15. The sample of monoatomic gas undergoes a process as represented by P – V graph (if $P_0V_0 = 1/3\ RT_0$) then



(P) $W_{1\to2} = 1/3RT_0$ (Q) $Q_{1\to2\to3} = 11/6RT_0$ (R) $U_{1\to2} = RT_0/2$ (S) $W_{1\to2\to3} = 1/3RT_0$

Which of the following options are correct

(a) P, Q, R, S are correct (b) Only P, Q are correct

(c) Only R, S are correct (d) Only P, R, S correct

SOLUTION:

$$\omega_{1-2} = P_0V_0 = \frac{1}{3}RT_0$$

$$Q_{2\to 3} = nC_V\Delta T = \frac{f}{2}2V_0P_0 = RT_0$$

$$Q_{1\to 2} = nC_P\Delta T = \frac{5}{6}RT_0$$

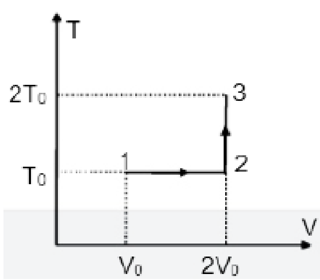
$$\omega = \frac{1}{3}RT_0$$

$$U_{1-2} = nC_V\Delta T$$

$$= n\frac{3}{2}RT_0$$

$$= \frac{RT_0}{2}$$

16. A sample of monoatomic gas undergoes a process as represented by T – V graph (if $P_0V_0 = 1/3\ RT_0$) then



(P) $W_{1\to 2} = \frac{1}{3}RT_0\ell n 2$ (Q) $Q_{1\to 2\to 3} = \frac{RT_0}{6}\left(2\ell n(2)+3\right)$

(R) $U_{1\to 2} = 0$ (S) $W_{1\to 2\to 3} = \frac{RT_0}{3}\ell n 2$

Which of the following option are correct:

(a) P, Q are incorrect (b) R, S are incorrect (c) P, Q, S are incorrect (d) none of these

Solution:

$$\omega_{1-2} = nRT_0 \ln 2$$

$$Q_{1-2-3} = Q_{12} + Q_{23}$$

$$= d\omega_{12} + dU_{2-3}$$

$$= \frac{RT_0}{3} \ell n\ _2 + n\frac{f}{2} RT_0$$

$$= \frac{RT_0}{3} \ell n\ _2 + \frac{1}{3}\frac{3}{2} RT_0$$

$$\omega_{1-2-3} = \frac{1}{3} R_0 T_0\ \ \ell n\ _2$$

17. Length of string of a musical instrument is varied from L_o to $2L_o$ in 4 different cases. Wire is made of different materials of mass per unit length μ, 2μ, 3μ, 4μ respectively. For first case (string – 1) length is L_o, Tension is T_o then fundamental frequency is f_o, for second case length of the string is $\frac{3L_o}{2}$ (3rd Harmonic), for third case length of the string is $\frac{5L_o}{4}$ (5th Harmonic) and for the fourth case length of the string is $\frac{7L_o}{4}$ (14th harmonic). If frequency of all is same then tension in strings in terms of T_o will be:

(a) String – 1 (P) T_o

(b) String – 2 (Q) $\frac{T_o}{\sqrt{2}}$

(c) String – 3 (R) $\frac{T_o}{2}$

(d) String – 4 (S) $\frac{T_o}{16}$

(T) $\frac{3T_o}{16}$

Solution:

(1) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $L = \frac{3L_0}{2}$

$$f_2 = \frac{3}{2\frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}}$$

$$T_2 = \frac{T_0}{2}$$

(3) $L = \frac{5L_0}{4}$

$$T_3 = \frac{T_0}{16}$$

(4) Similarly $T_4 = \frac{T_0}{16}$

<u>SECTION – 3</u>

18. The free length of all four string is varied from L_0 to $2L_0$. Find the maximum fundamental frequency of 1, 2, 3, 4 in terms of f_0 (tension is same in all strings)

(a) String – 1	(P) 1
(b) String – 2	(Q) $\frac{1}{2}$
(c) String – 3	(R) $\frac{1}{\sqrt{2}}$
(d) String – 4	(S) $\frac{1}{\sqrt{3}}$
	(T) $\frac{1}{16}$
	(U) $\frac{3}{16}$

<u>Solution:</u>

(1) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $$f_2 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{f_0}{\sqrt{2}}$$

$$f_3 = \frac{1}{L_0}\sqrt{\frac{T_2}{3\mu}} = \frac{f_0}{\sqrt{3}}$$

$$f_4 = \frac{1}{L_0}\sqrt{\frac{T_2}{4\mu}} = \frac{f_0}{2}$$

CHEMISTRY – JEE ADVANCED PAPER – 2

SECTION – 1

1. With reference to aqua regia, choose the correct option(s):

   (a) Aqua regia is prepared by mixing conc. HCl and conc. HNO_3 in 3: 1 molar ratio.

   (b) Reaction of gold with aqua regia produces an anion having Au in +3 oxidation state.

   (c) Reaction of gold with aqua regia produces NO_2 in the absence of air

   (d) The yellow colour of aqua regia is due to the presence of NOCl & Cl_2.

Solution::

A, B, D

$$Au + HNO_3 + 4HCl[AuCl_4]^- + [NO] + H_3O^+ + H_2O$$

a) Aqua regia is HCl & HNO_3 (conc.) in a 3 : 1

b) Oxidation state of Au in $[AuCl_4]^-$ is +3.

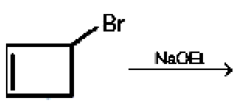
c) NOCl/NO is formed

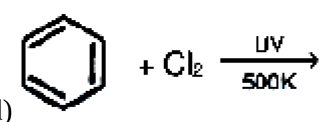
d) NOCl is yellow in colour

2. Choose the correct option that gives aromatic compound as major product:

(a) 

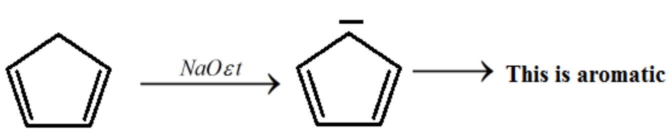
(b) 

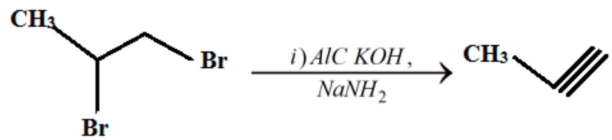
(c) 

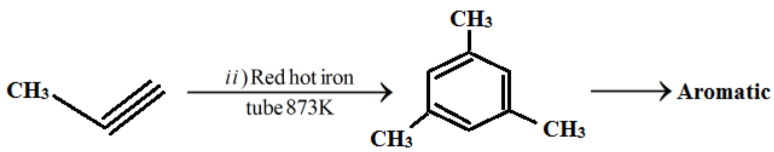
(d) 

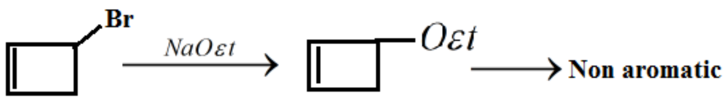
Solution::

A, B

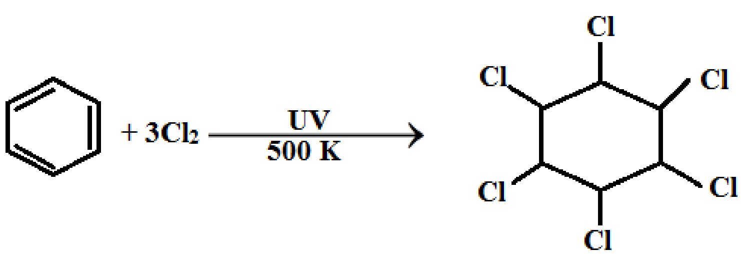
(a) 

(b) 

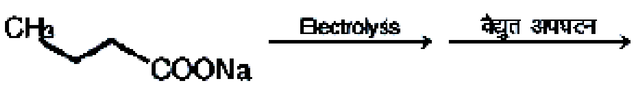


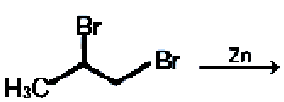
(c) 

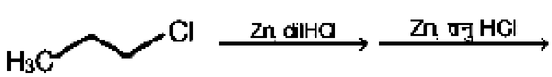
This will not give  as it is anti aromatic.

(d) 

3. Which of the following reaction produce propane as major product?

(a) 

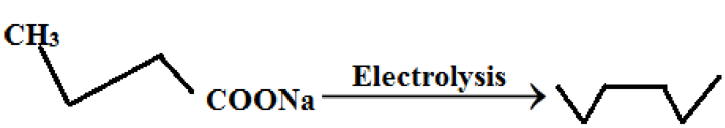
(b) 

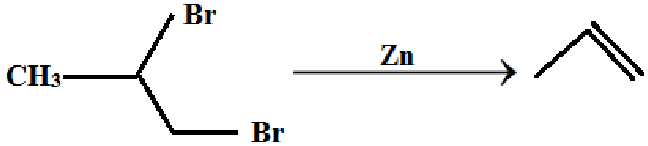
(c) 

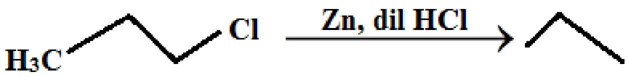
(d) 

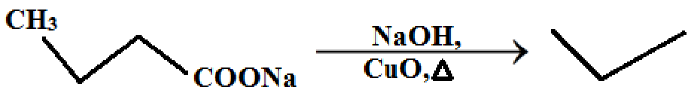
Solution:

(C, D)

(A) 

(B) 

(C) 

(D) 

4. Which of the following is/are correct

(a) Teflon is formed by polymerization of tetrafluoroethene.

(b) Natural rubber is the trans from of polyisoprene.

(c) Cellulose contains only α-D-glucose linkage

(d) Nylon-6 contains amide linkage.

Solution:

(A, D)

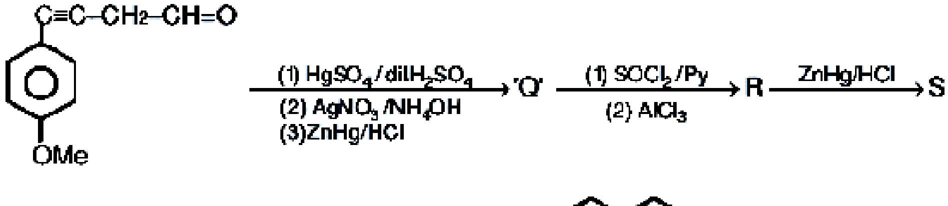
A) Fact.

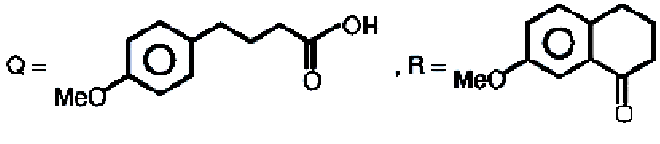
B) Natural rubber is Cis form of polyisoprene

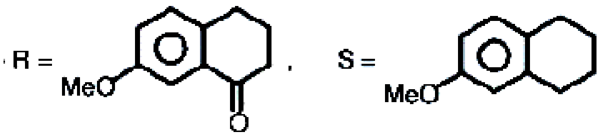
C) Cellulose contains B – 1, 4 – glycosidic linkage

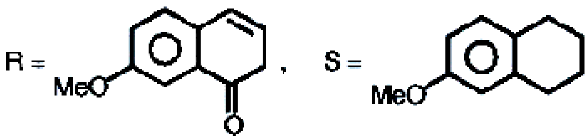
D) Nylon 6 contains amide linkage.

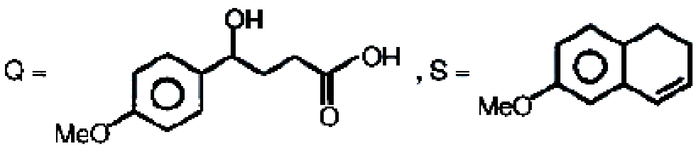
5.



(a) 

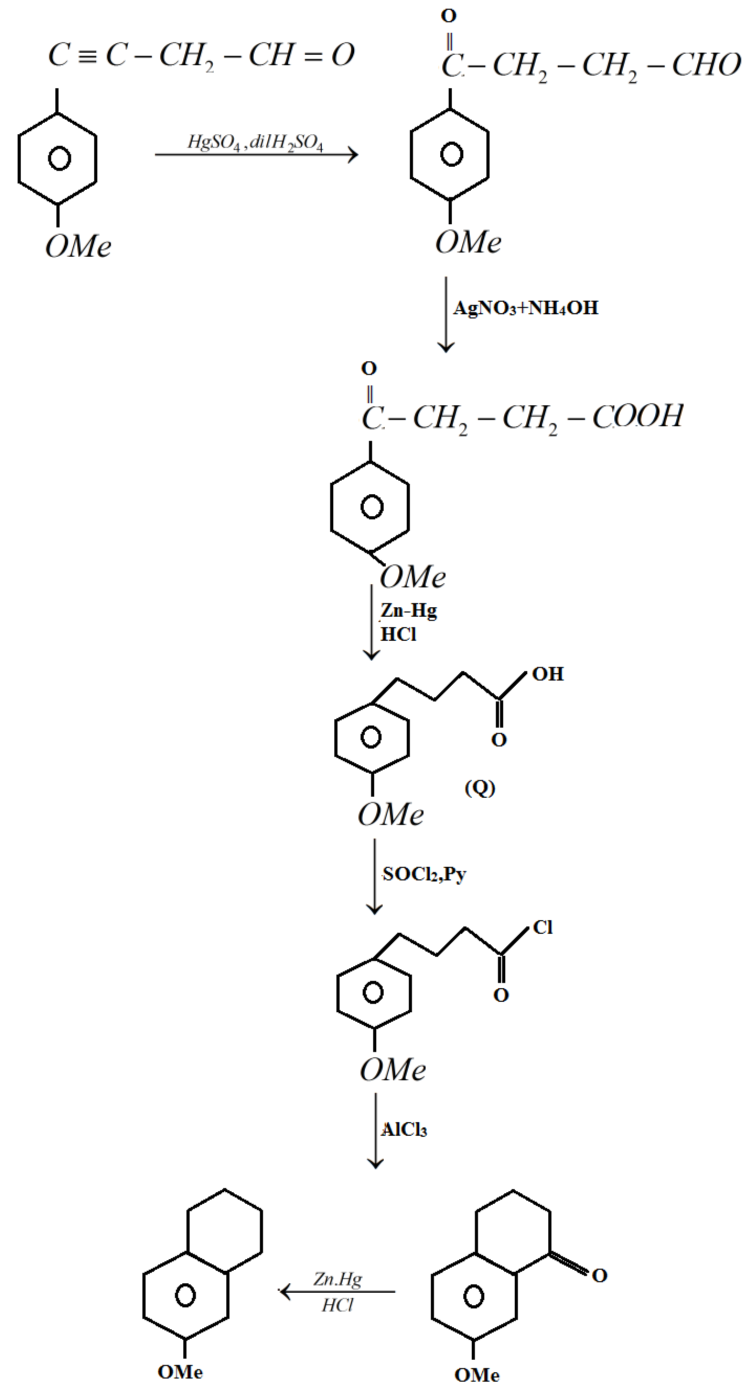
(b) 

(c) 

(d) 

Solution:

(A, B)



6. Consider the following reaction (unbalanced)

Zn + Hot conc. $H_2SO_4 \rightarrow G + R + X$

Zn + conc. $NaOH \rightarrow T + Q$

$G + H_2S + NH_3\,(aq) \rightarrow Z$ (precipitate) + X + Y

Choose the correct option(s)

(a) R is a V-shaped molecule

(b) Z is dirty white in colour

(c) Bond order of Q is 1 in its ground state (d) The oxidation state of Zn in T is +1.

Solution:

A, B, C

$$Zn + \underset{(conc)}{H_2SO_4} \rightarrow \underset{G}{ZnSO_4} + \underset{R}{SO_2} + \underset{X}{H_2O}$$

$$Zn + conc.\,NaOH \rightarrow \underset{T}{Na_2ZnO_2} + \underset{Q}{H_2}$$

$$\underset{G}{ZnSO_4} + H_2S + NH_{3(aq)} \rightarrow \underset{(Z)}{ZnS} \downarrow + \underset{(Y)}{(NH_4)_2SO_4} + \underset{(X)}{H_2O}$$

A) SO_2 is v shaped.

B) ZnS is dirty white in colour.

C) Bond order of H_2 is 1.

D) Oxidation state of Zn in Na_2ZnO_2 is +2.

7. In the Mac. Arthur process of extraction

$$Au \xrightarrow{NaCN+Q} R \xrightarrow[with\ Y]{extracted} Z$$

(a) R is $[Au(CN_4)]^{(-)}$ (b) Z is $[Zn(CN)_4]^{2-}$ (c) Q is O_2 (d) Y is Zn

Solution:

B, C, D (from text book).

8. For He^+ the electron is in orbit with energy equal to 3.4eV. The azimuthal quantum number for that orbit is 2 and magnetic quantum number is 0. Then which of the following is/are correct.

(a) The subshell is 4d.

(b) The number of angular nodes in it is 2.

(c) The numbers of radial nodes in it is 3.

(d) The nuclear charge experienced in n = 4 is 2e less than that in n = 1, where e is electric charge.

Solution::

A, B

$$E = E_0 \frac{z^2}{n^2}$$

$$3.4 = 13.6 \times \frac{4}{n^2}$$

n = 4

l = 2

a) Subshell is 4 d

b) Number of angular nodes is 2

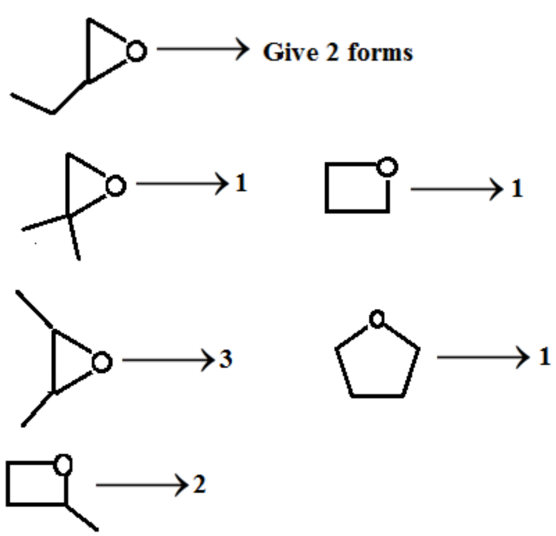
c) Number of radial nodes is 1.

d) Nuclear charge would be the same.

SECTION – 2

9. Calculate the total number of cyclic ether (including stereo) having formula C_4H_8O

Solution:



Total 10

10. 1 mole of Rhombic sulphur is treated with conc. HNO_3. Find the mass of H_2O formed.

Solution:

$$S_8 + HNO_3 \rightarrow H_2SO_4 + NO_2 + H_2O$$

Balancing

$$S_8 + 48HNO_3 \rightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$$

∴ Mass of H_2O = 288

11. Mole fraction of urea in 900 gram water is 0.05. Density of Solution is 1.2 g/cm^3. Find molarity of Solution.

Solution:

No of moles of H_2O $= \frac{900}{18} = 50$

$$\therefore \frac{n_1}{n_1 + 50} = 0.05$$ (n_1 is No. of moles of urea)

$$\Rightarrow n_1 = 2.63$$

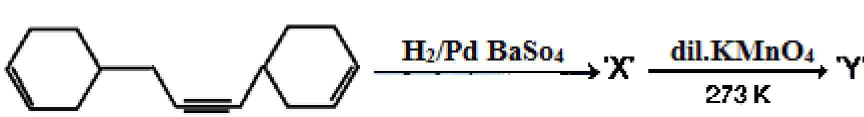
Weight of urea = 2.63 × 60 = 157.8 g

Total weight = 157.8 + 900 = 1057.8g

$$\therefore Volume = \frac{1057.8}{1.2} = 881.5\,cm^3$$

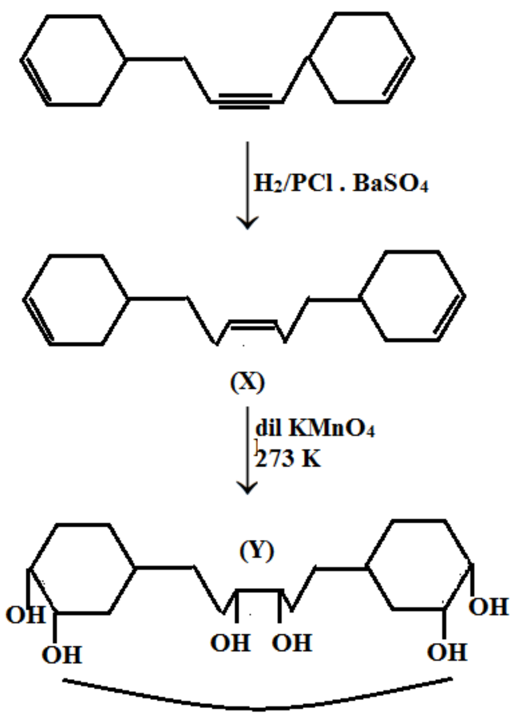
$$\therefore Molarity = \frac{2.63}{0.8} = 2.99$$

12. Number of hydroxyl group in compound 'Y' is:



Solution:

(6)



Total 6 – OH groups.

13. In following reaction the value of K is $5 \times 10^{-4}\ S^{-1}$.

$$2N_2O_5 \xrightarrow{\Delta} 2N_2O_4 + O_2$$

Initial pressure was 1 atm, while the final pressure was 1.45 atm at time y × 10^3 sec calculate 'y'.

Solution::

From unit of K reaction is first order.

$$2N_2O_5 \xrightarrow{\Delta} 2N_2O_4 + O_2$$

t = 0	1	0	0
t = t	1 – P	P	P/2
t = ∞	0	1	0.5

P_0 = 1 atm, P_t = 1.45 atm, P_∞ = 1.5 atm

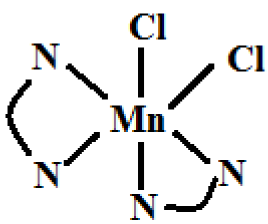
$$t = \frac{1}{2K} \ln\left(\frac{P_\infty - P_0}{P_\infty - P_1}\right)$$

$$= 2.3 \times 10^3$$

$$\Rightarrow y = 2.3$$

14. Number of N-Mn-Cl bonds [N-Mn bonds is cis to Mn-Cl bond] in cis $[Mn(en)_2Cl_2]$ are …….

Solution:



→ This is the Cis form of $[Mn(en)_2Cl_2]$

∴ No of N – Mn – Cl bonds = 6

SECTION – 3

Match the column

	List 1		List 2
P	Radius	I	$I \propto n^{-1}$
Q	Angular momentum	II	$II \propto n^{-2}$
R	Kinetic energy	III	$III \propto n^{-0}$
S	Potential energy	IV	$IV \propto n^1$
			$V \propto n^2$

15. Which of the following is correct

(a) P I (b) P II (c) P V (d) P III

Solution:

$$r_n = 0.529\left(\frac{n^2}{Z}\right) A^\circ \Rightarrow r_n \propto n^2$$

(C)

16. Which of following is correct.

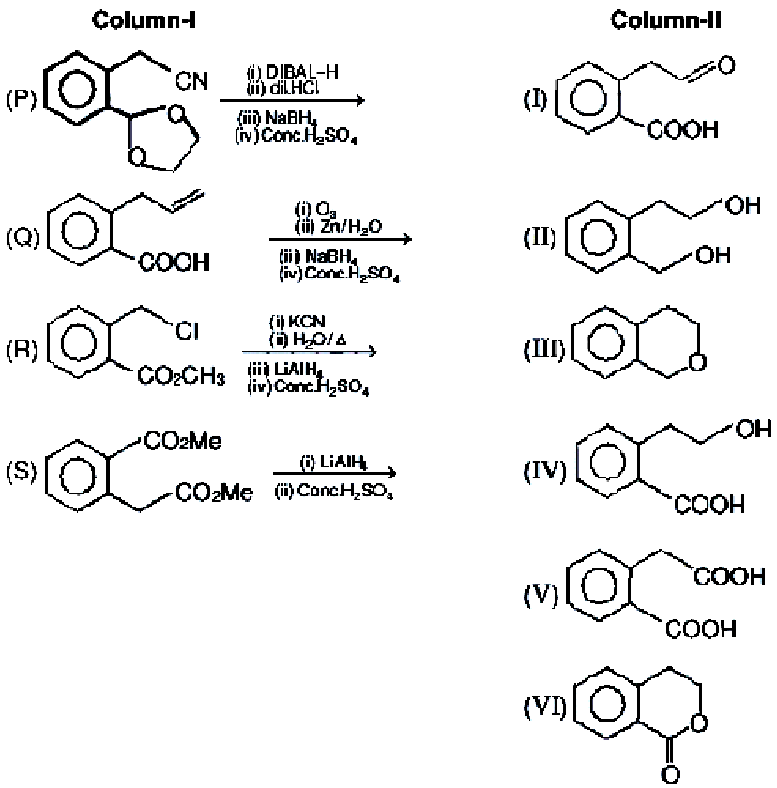
(a) S IV (b) R I (c) R II (d) S III

Solution:

$$K.E \propto \frac{Z^2}{n^2}$$

(C)

Answer the question no. 17 & 18 on the basis of information given in Column – I & Column – II. Match the reactant in column – I with the possible intermediates and products of Column – II.

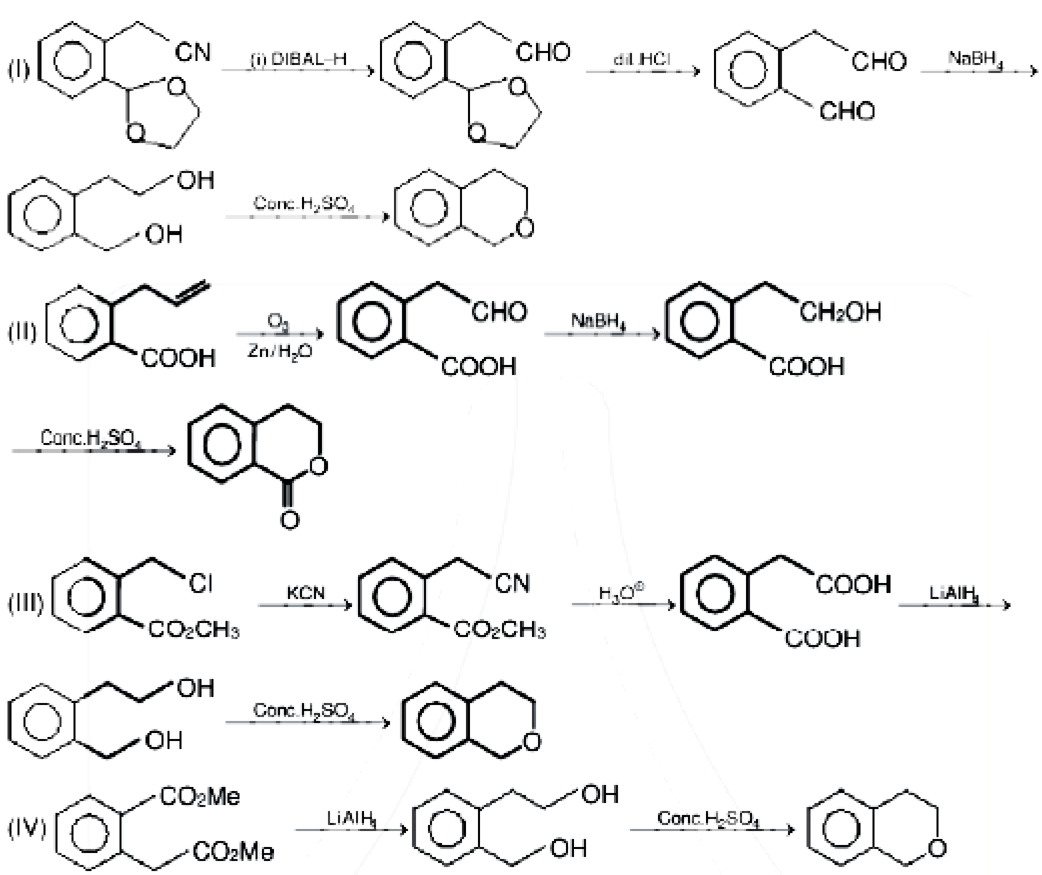


17. Which of the following is correct?

(a) P – II, III; S – II, III (b) P – II, IV; S – II, III (c) P – III, VI; S – II, III (d) P – I, III; S – IV, V

Solution::

(a)



18. Which of the following is correct?

(a) Q – I, IV, VI; R – II, III, V

(b) Q – I, III, VI; R – II, IV, V

(c) Q – I, II, VI; R – II, III, VI

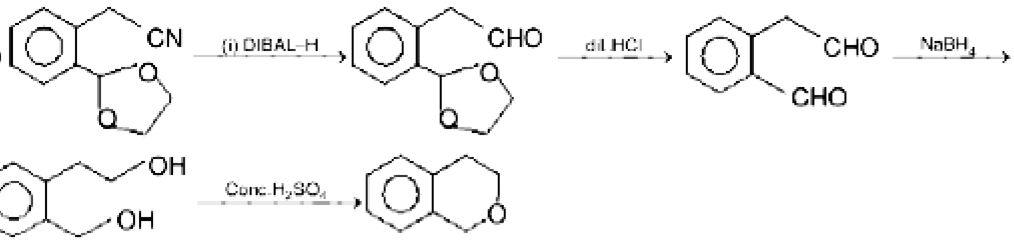
(d) Q – I, IV, V; R – III, I, V

Solution::

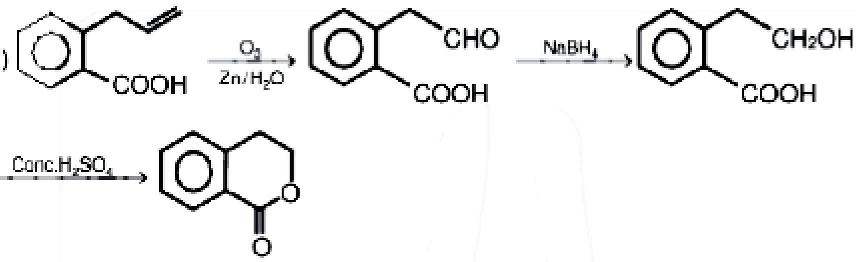
(a)

CHEMISTRY – JEE ADVANCED PAPER – 2

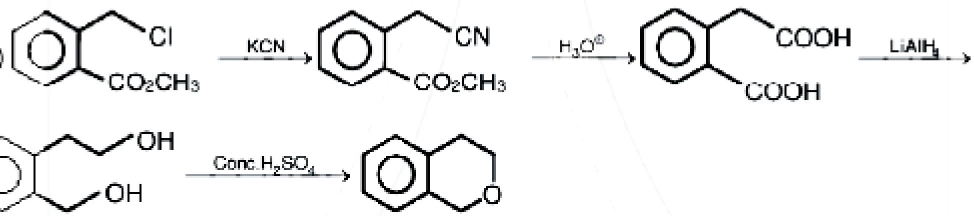
(I)



(II)



(III)



(IV)