JEE ADVANCED PAPER-I

Time Duration: 3 Hours **Maximum Marks : 183**

INSTURCTIONS:

QUESTION PAPER FORMAT AND MARKING SCHEME :

1. The question paper has **three parts** : Physics, Chemistry and Mathematics.
2. Each part has three sections as detailed in the following table :

Section	Question Type	Number of Questions	Category-wise Marks for Each Question				Maximum Marks of the Section
			Full Marks	Partial Marks	Zero Marks	Negative Marks	
1	One or more correct option(s)	7	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	–2 In all other cases	28
2	Single digit Integer (0-9)	5	+3 If only the bubble corresponding to the correct answer is darkened	—	0 In all other cases	—	15
3	Single Correct Option	6	+3 If only the bubble corresponding to the correct option is darkened	—	0 If none of the bubbles is darkened	–1 In all other cases	18

SECTION - 1 (Maximum Marks : 28)

This section contains **SEVEN** questions.

Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.

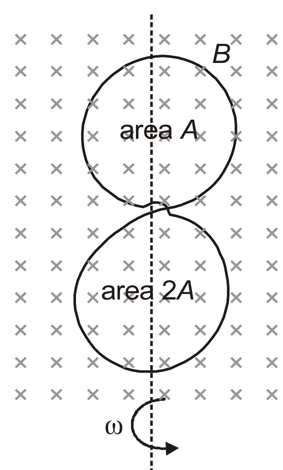
For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get –2 marks, as a wrong option is also darkened.

1. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field B points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(A) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

(B) The emf induced in the loop is proportional to the sum of the areas of the two loops

(C) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(D) The net emf induced due to both the loops is proportional to cos ωt

Answer (A, C)

Sol. $\phi = BA\cos\omega t$

$|e| = B\omega\sin\omega t$

For loop 1

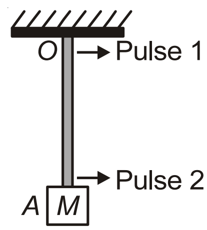
$\phi_1 = BA\cos\omega t$: $|e_1| = AB\omega\sin\omega t$

$\phi_2 = 2BA\sin\omega t$ $|e_2| = 2BA\omega\sin\omega t$

e_1 and e_2 oppose each other, so amplitude of net emf induced = $2BA\omega - BA\omega = BA\omega$

e_1 and e_2 will be peak at $\omega t = \pi/2$ or $\theta = 90°$.

2. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(A) The velocity of any pulse along the rope is independent of its frequency and wavelength

(B) The time $T_{AO} = T_{OA}$

(C) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(D) The wavelength of Pulse 1 becomes longer when it reaches point A

Answer (A, B, C)

Sol. At any point $v = \sqrt{\dfrac{T}{\mu}}$, since velocity depends on tension at a point and mass per unit length, so time from O to A and A to O will be same.

$T_{OA} = T_{AO}$

Also frequency remain constant at all point. To keep it same, $\dfrac{v}{\lambda}$ must be same and v is maximum at $O(T \rightarrow \text{maximum})$ so λ must be longest at O.

3. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true?

(A) The plate will continue to move with constant non-zero acceleration, at all times

(B) The resistive force experienced by the plate is proportional to v

(C) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(D) At a later time the external force F balances the resistive force

Answer (B, C, D)

Sol. n = number of molecules per unit volume

u = average speed of gas molecules

When plate is moving with speed v, relative speed of molecules w.r.t. leading force = $v + u$

Coming head on, momentum transformed to plate per collision = $2m(u + v)$

Number of collision in time $\Delta t = \frac{1}{2}(v + v_0)n\Delta tA$

where A = Surface area

So, momentum transferred in time Δt = $m(u + v)^2 nA\Delta t$ from front surface

Similarly momentum transferred in time = $m(u - v)^2 nA\Delta t$ from back surface.

Net force = $mnA[(v + u)^2 - (u - v)^2]$

$= mnA[4vu]$

$F \propto v$

$P_{\text{leading}} - P_{\text{trailing}} = mn[u + v]^2 - mn[u - v]^2$

$= mn[4uv] = 4mnuv$

$\Delta P \propto uv$

4. A human body has a surface area of approximately 1m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of $\sigma T_0^4 = 460 \text{ Wm}^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct?

(A) The amount of energy radiated by the body in 1 second is close to 60 Joules

(B) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

(C) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(D) Reducing the exposed surface area of the body (*e.g.* by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

Answer (A, C, D)

Sol. Body metabolic system will have to produce additional energy in order to maintain the temperature.

Let due to internal metabolism, power produced is I,

As temperature of body is constant, $\Delta Q_{net} = 0$

$\Rightarrow \quad I = \sigma A\,[T^4 - T_0^4]$

$\Rightarrow \quad dI = 4\sigma A T_0^3\ (dT_0) = 4\sigma A T_0^3 . \Delta T_0$ (option C)

$= 4 \times 460 \times \frac{10}{300} \approx 60$ J/s (option A)

5. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(A) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(D) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

Answer (A, C, D)

Sol. $\delta_m = (2i) - A$

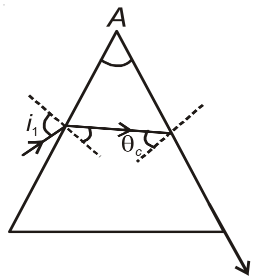
$\Rightarrow$ $2A = 2i$

$\Rightarrow$ $i = A$ and $r = A/2$ (look solution at right side)

$$\mu = \frac{\sin\left[\frac{A+A}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$

$$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\mu = 2\cos\frac{A}{2}$$



$$1\sin i_1 = \mu \times \sin(A - \theta_C)$$

$$= 2\cos\frac{A}{2}[\sin A\cos\theta_C - \cos A\sin\theta_C]$$

$$= 2\cos\frac{A}{2}\left[\sin A\sqrt{1-\sin^2\theta_C} - \cos A\frac{1}{\mu}\right]$$

$$= 2\cos\frac{A}{2}\left[\sin A\sqrt{1-\frac{1}{\mu^2}} - \frac{\cos A}{\left(2\cos\frac{A}{2}\right)}\right]$$

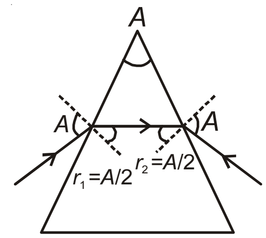
$$= 2\cos\frac{A}{2}\left[\sin A\sqrt{1-\frac{1}{4\cos^2\frac{A}{2}}} - \frac{\cos A}{2\cos\frac{A}{2}}\right]$$

$$i_1 = \sin^{-1}\left[\sin A\left[\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right]\right]$$

$r_1 = \frac{A}{2}$ for minimum deviation.

$$\frac{A}{2} = \cos^{-1}\left[\frac{\mu}{2}\right] \quad \Rightarrow \quad A = 2\cos^{-1}\left[\frac{\mu}{2}\right]$$

Calculation of r for $i = A$



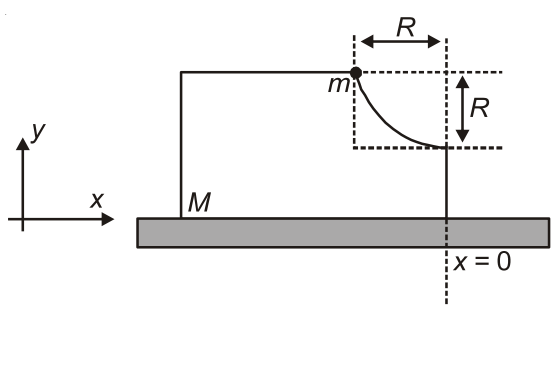
$$1\sin A = \mu\sin r$$

$$\sin A = 2\cos\frac{A}{2}.\sin r$$

$$\sin r = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos\frac{A}{2}} = \sin\frac{A}{2}$$

$$\Rightarrow r = \frac{A}{2}$$

6. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



(A) The x component of displacement of the center of mass of the block M is: $-\dfrac{mR}{M+m}$

(B) The velocity of the point mass m is: $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(C) The position of the point mass is: $x = -\sqrt{2}\dfrac{mR}{M+m}$

(D) The velocity of the block M is: $V = -\dfrac{m}{M}\sqrt{2gR}$

Answer (A, B)

Sol. v = velocity of M

u = velocity of m

$mu = -MV$ ...(i)

$mgR = \frac{1}{2}mu^2 + \frac{1}{2}MV^2$ ...(ii)

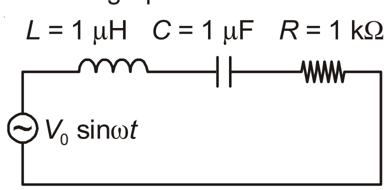
$\therefore \boxed{u = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}} \quad v = \dfrac{-m}{M}\sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}}$

and $Mx' = m(R - x')$ where x' = displacement of block M

$x'(M + m) = mR \quad \Rightarrow \quad x' = \dfrac{mR}{m+M}$ towards left

Or $\boxed{x' = -\dfrac{mR}{m+M}}$

7. In the circuit shown, $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 1\ k\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(A) The frequency at which the current will be in phase with the voltage is independent of R

(B) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}

(C) At $\omega >> 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

(D) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

Answer (A, D)

Sol. At $\omega >> 10^6$

$X_L = \omega L$

$= 10^6 \times 10^{-6}$ for $\omega = 10^6$

$= 1\ \Omega \Rightarrow X_L >> 1\ \Omega$ for $\omega >> 10^6$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{10^6 \times 10^{-6}} = 1\,\Omega$ at $\omega = 10^6 \Rightarrow X_c << 1\Omega$ for $\omega >> 10^6$

$R = 1\ k\Omega$

$$\omega_R = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}10^{-6}}} = 10^6 \text{ rad.s}^{-1}$$

At $\omega \to 0$ $X_c \to \infty \Rightarrow$ current $\to 0$

SECTION - 2 (Maximum Marks : 15)

This section contains **FIVE** questions.

The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

For each question, darken the bubble corresponding to the correct integer in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	In all other cases.

8. A drop of liquid of radius $R = 10^{-2}$m having surface tension $S = \frac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is

Answer (6)

Sol. Let radius of small drops = r.

$$\frac{4}{3}\pi R^3 = K\frac{4}{3}\pi r^3$$

$$R = K^{\frac{1}{3}} r \qquad \text{...(i)}$$

$S(K4\pi r^2 - 4\pi R^2) = 10^{-3}$

$$\frac{0.1}{4\pi} 4\pi \left(k\frac{R^2}{K^{\frac{2}{3}}} - R^2 \right) = 10^{-3}$$

$$R^2\left(K^{\frac{1}{3}} - 1\right) = 10^{-2}$$

$$K^{\frac{1}{3}} - 1 = 100$$

$$K^{\frac{1}{3}} = 101$$

$$10^{\frac{\alpha}{3}} = 101$$

$\alpha \approx 6$

9. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is *reflected* by a large car *approaching* the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency *it* has received).

Answer (6)

Sol. Frequency of sound emitted by stationary source, f_0 = 492 Hz

Frequency of sound reflected by moving car

$$f' = f_0\left[\frac{c + v_c}{c - v_c}\right]$$

where c = speed of sound in air = 330 m/s

v_c = speed of car = 2 m/s

On solving

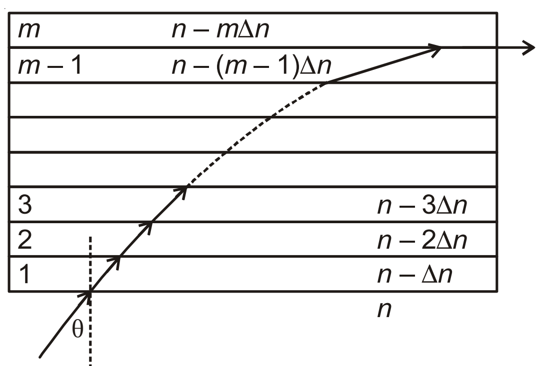
$$f' = 492 \times \frac{332}{328} \simeq 498 \text{ Hz}$$

Beat frequency of resulting signal = $f' - f_0$

= (498 – 492) Hz

= 6 Hz

10. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle θ = 30°. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and Δn = 0.1 (see the figure). The ray is refracted out parallel to the interface between the $(m - 1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m?



Answer (8)

Sol. Considering Snell's law between first layer and mth layer.

$$n\sin\theta = (n - m\Delta n)\sin 90^\circ$$

$$1.6 \times \frac{1}{2} = (1.6 - m(0.1))1$$

$$m = \frac{0.8}{0.1} = 8$$

11. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f}$ = 6.25, then the smallest possible n_f is

Answer (5)

Sol. $\text{PE} = -27.2\frac{Z^2}{n^2}$ $\quad$ Z = 1 for H

$$\text{PE} = \frac{-27.2}{n^2}$$

$$V_i = \text{PE}_1 = \frac{-27.2}{n_i^2} \qquad \therefore \quad \text{PE}_2 = \frac{-27.2}{n_f^2} = v_f$$

$$\frac{V_i}{V_f} = \frac{n_f^2}{n_i^2} = \left(\frac{n_f}{n_i}\right)^2 = 6.25$$

$n_f = 2.5\ n_i$

$$\frac{n_f}{n_i} = \frac{5}{2}$$

$\Rightarrow\ n_i \text{ min} = 2$

$n_f \text{ min} = 5$

$n_f = 5$

12. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately
(you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$).

Answer (5)

Sol. Suppose total volume of blood is V ml

$\because$ 2.5 ml of blood activity = 115 Bq

$\therefore$ 1 ml of blood activity = $\dfrac{115}{2.5}$

$\therefore$ Activity of whole blood = $\left(\dfrac{115}{2.5}\right)$V Bq

$$\left(\frac{115V}{2.5}\right) = 2.4\times10^5 e^{-\lambda t}$$

$$= 2.4\times10^5 e^{-\frac{\ln 2}{8\times24}\times11.5}$$

$$= 2.4\times10^5 e^{-\frac{0.7\times11.5}{8\times24}}$$

$$= 2.4\times10^5 e^{-\frac{1}{24}}$$

$$= 2.4\times10^5 \left(1-\frac{1}{24}\right)$$

$$V = 2.4\times10^5 \times\frac{23}{24}\times\frac{2.5}{115} \approx 5000 \text{ ml}$$

$\therefore$ Total volume of blood = 5 L

SECTION - 3 (Maximum Marks : 18)

This section contains **SIX** questions of matching type.

This section contains **TWO** tables (each having 3 columns and 4 rows)

Based on each table, there are **THREE** questions.

Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–1	In all other cases.

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

13. In which case will the particle describe a helical path with axis along the positive z direction?

(A) (III) (iii) (P)
(B) (IV) (i) (S)
(C) (II) (ii) (R)
(D) (IV) (ii) (R)

Answer (B)

Sol. Proton will move in helical path with axis along positive z-direction

If $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$, $\vec{E} = E_0\hat{z}$ and $\vec{B} = B_0\hat{z}$

14. In which case will the particle move in a straight line with constant velocity?

(A) (III) (ii) (R)
(B) (II) (iii) (S)
(C) (IV) (i) (S)
(D) (III) (iii) (P)

Answer (B)

Sol. Electron will move in straight line with constant velocity if

$$\vec{v} = \frac{E_0}{B_0}\hat{y},\ \vec{E} = -E_0\hat{x},\ \vec{B} = B_0\hat{z}$$

15. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$)?

(A) (III) (ii) (P)
(B) (II) (iii) (Q)
(C) (IV) (ii) (S)
(D) (III) (ii) (R)

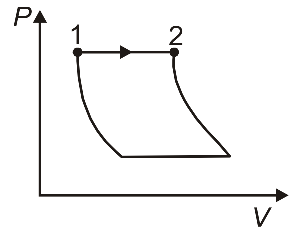
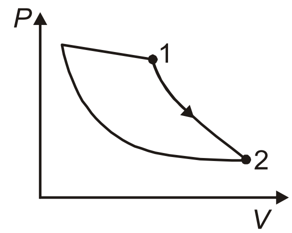
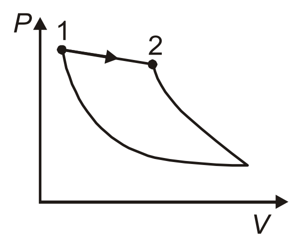
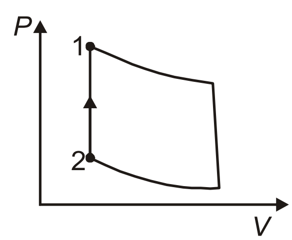
Answer (D)

Sol. Proton will move in straight line along negative y-direction when

$\vec{v} = 0$, $\vec{E} = -E_0\hat{y}$ and $\vec{B} = B_0\hat{y}$

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding *P*-*V* diagrams in column 3 of the table. Consider only the path from state 1 to state 2. *W* denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is *n*.

Column 1	**Column 2**	**Column 3**
(I) $W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which one of the following options is the correct combination?

(A) (IV) (ii) (S) (B) (III) (ii) (S)

(C) (II) (iv) (P) (D) (II) (iv) (R)

Answer (B)

Sol. Option (B) is correct combination.

$W_{1\to2} = 0 \quad \Rightarrow \quad$ Isochoric process

17. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iii) (S) (B) (III) (iii) (P)

(C) (II) (iii) (P) (D) (II) (iv) (R)

Answer (C)

Sol. Option (C) is correct representation.

$$W_{1 \to 2} = -PV_2 + PV_1 \quad \Rightarrow \quad \text{Isobaric}$$

18. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(A) (IV) (ii) (R)

(B) (I) (iv) (Q)

(C) (I) (ii) (Q)

(D) (III) (iv) (R)

Answer (B)

Sol. Adiabatic process is used as a correction in the determination of the speed of sound in an ideal gas.

$\therefore$ Correct option is (B)

Work done on the system in adiabatic process = $\dfrac{-[P_2V_2 - P_1V_1]}{\gamma - 1}$

SECTION - 1 (Maximum Marks : 28)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–2	In all other cases

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

19. An ideal gas is expanded from (p_1, V_1, T_1) to (p_2, V_2, T_2) under different conditions. The correct statement(s) among the following is(are)

(A) The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

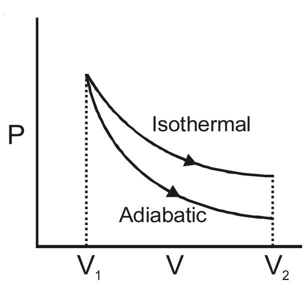
(B) The work done by the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions

(C) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

(D) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

Answer (B, C, D)

Sol. (B)



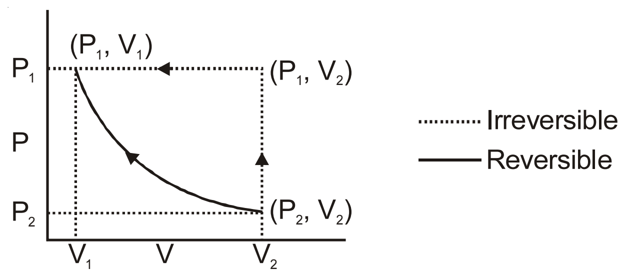
$W_{Isothermal} > W_{Adiabatic}$

(C) In free expansion, $P_{ex} = 0 \Rightarrow w = 0$

For isothermal free expansion of an ideal gas,

$\Delta U = 0 \Rightarrow q = 0 \therefore$ adiabatic also

(D) Area under irreversible P v/s V graph in compression is more than that under P-V graph of reversible compression.



20. The correct statement(s) about the oxoacids, $HClO_4$ and HClO, is(are)

(A) The conjugate base of $HClO_4$ is weaker base than H_2O

(B) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

(C) $HClO_4$ is formed in the reaction between Cl_2 and H_2O

(D) The central atom in both $HClO_4$ and HClO is sp^3 hybridized

Answer (A, B, D)

Sol. (A) $HClO_4 > H_2O$ Acidic character

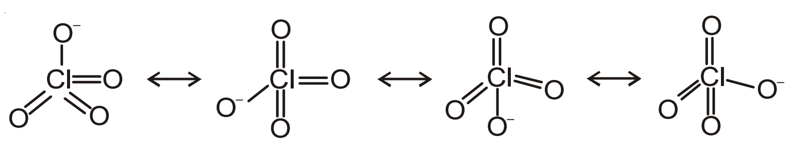
$ClO_4^- < OH^-$ Conjugate Base

Strong acids have weak conjugate base

(B) $HClO_4 >$ HClO Acidic strength

$HClO_4 \longrightarrow H^+ + ClO_4^-$

$HClO \longrightarrow H^+ + ClO^-$

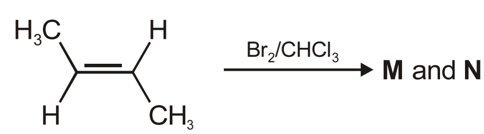


In ClO_4^-, nagative charge is dispersed on four oxygen, so better resonance stabilised.

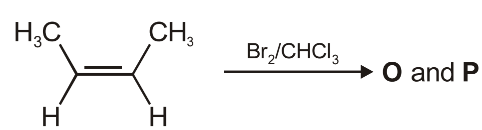
(D) Both $HClO_4$ and HClO central atom is sp^3 hybridised

21. The correct statement(s) for the following addition reactions is(are)

(i)



(ii)



(A) (**M** and **O**) and (**N** and **P**) are two pairs of enantiomers

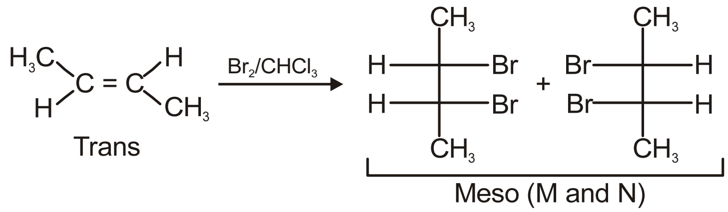
(B) Bromination proceeds through *trans*-addition in both the reactions

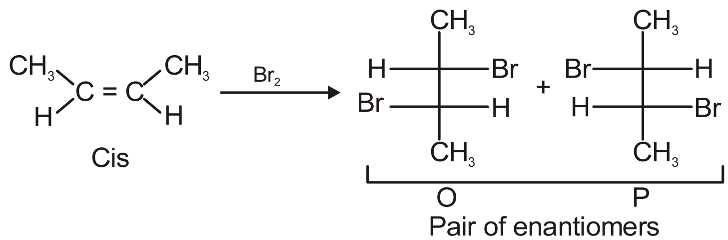
(C) (**M** and **O**) and (**N** and **P**) are two pairs of diastereomers

(D) **O** and **P** are identical molecules

Answer (B, C)

Sol.

(i) 

(ii) 

Cis anti Racemic

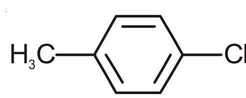
M and N are meso (identical)

O and P are pair of enantiomers

(B) Bromination proceeds through anti addition.

(C) (M and O) and (N and P) are two pairs of diastereomer.

22. The IUPAC name(s) of the following compound is(are)



(A) 4-methylchlorobenzene

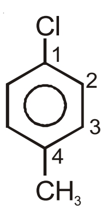
(B) 1-chloro-4-methylbenzene

(C) 4-chlorotoluene

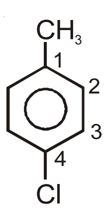
(D) 1-methyl-4-chlorobenzene

Answer (B, C)

Sol. IUPAC name

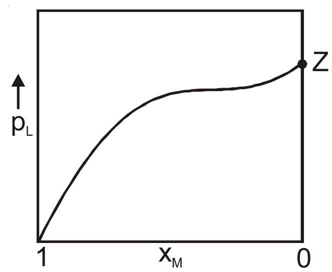
(B) 

1-chloro-4-methylbenzene

(C) 

4-chlorotoluene

23. For a solution formed by mixing liquids **L** and **M**, the vapour pressure of **L** plotted against the mole fraction of **M** in solution is shown in the following figure. Here x_L and x_M represent mole fractions of **L** and **M**, respectively, in the solution. The correct statement(s) applicable to this system is(are)



(A) The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed when $x_L \rightarrow 0$

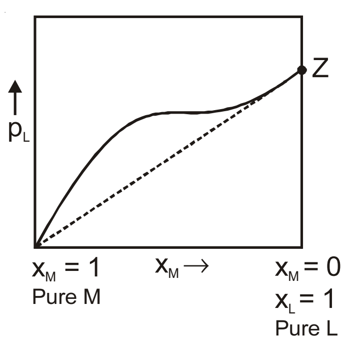
(B) The point **Z** represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \rightarrow 1$

(C) Attractive intermolecular interactions between **L-L** in pure liquid **L** and **M-M** in pure liquid M are stronger than those between **L-M** when mixed in solution

(D) The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$

Answer (B, C)

Sol.



Point Z represents v.p. of pure liquid L

At $x_L \rightarrow 1$, solution is very dilute, L becomes solvent. Very dilute solution of M in L is nearly ideal and obey Raoults law ($p_L = x_L p_L^{\circ}$)

Also, there is positive deviation indicated by graph above dotted line (expected for ideal solution)

$\therefore$ L-M interaction < L - L & M - M Interactions.

24. Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2 \cdot 6H_2O$(**X**) and NH_4Cl gives an octahedral complex **Y** in the presence of air. In aqueous solution, complex **Y** behaves as 1:3 electrolyte. The reaction of **X** with excess HCl at room temperature results in the formation of a blue coloured complex **Z**. The calculated spin only magnetic moment of **X** and **Z** is 3.87 B.M., whereas it is zero for complex **Y**. Among the following options, which statement(s) is(are) correct?

(A) **Z** is a tetrahedral complex

(B) When **X** and **Z** are in equilibrium at 0°C, the colour of the solution is pink

(C) The hybridization of the central metal ion in **Y** is d^2sp^3

(D) Addition of silver nitrate to **Y** gives only two equivalents of silver chloride

Answer (A, B, C)

Sol. (**X**) = $CoCl_2 \cdot 6H_2O$, or, $[Co(H_2O)_6]Cl_2$ (Pink)

Co^{2+} = [Ar] $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ 3*d*

H_2O is weak field ligand, therefore No pairing of electron

$\therefore$ Number of unpaired electron, n = 3

$\Rightarrow \quad \mu_s = \sqrt{n(n+2)}$ B.M.

$= \sqrt{15}$ B.M.

= 3.87 B.M.

$$[Co(H_2O)_6]^{2+} + 6NH_{3(aq)} \rightleftharpoons [Co(NH_3)_6]^{2+} + 6H_2O$$

O_2 will oxidise $[Co(NH_3)_6]^{2+}$ to $[Co(NH_3)_6]^{3+}$, therefore shift in forward

$\therefore$ **Y** = $[Co(NH_3)_6]^{3+}Cl_3$ [1 : 3 complex]

Co(III) = [Ar] $\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ 3*d*

NH_3 is strong field ligand

$\therefore$ Forces electrons to pair up

$\therefore$ n = 0

μ = 0 B.M.

$[Co(NH_3)_6]^{3+}$ = [Ar] $\underbrace{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\times\times}\boxed{\times\times}}_{3d}\ \underbrace{\boxed{\times\times}}_{4s}\ \underbrace{\boxed{\times\times}\boxed{\times\times}\boxed{\times\times}}_{4p}$

d^2sp^3

Also, $\underset{\substack{\textbf{(X)}\\ \text{Pink}}}{[Co(H_2O)_6]^{2+}} + 4Cl^- \rightleftharpoons \underset{\substack{\textbf{(Z)}\\ \text{Blue}}}{[CoCl_4]^{2-}_{(aq)}} + 6H_2O$; ΔH = +ve

At 0°C, equilibrium shifts in backward, therefore Pink colour.

25. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

(A) Decrease in π^*-σ^* gap down the group

(B) The physical state of X_2 at room temperature changes from gas to solid down the group

(C) Decrease in HOMO-LUMO gap down the group

(D) Decrease in ionization energy down the group

Answer (A, C)

Sol. The colours arise from the absorption of light on promoting an electron from the ground state to higher state. On descending the group, the energy levels become closer and gap between HOMO – LUMO decreases.

HOMO is π^*

LUMO is σ^*

SECTION - 2 (Maximum Marks : 15)

This section contains **FIVE** questions.

The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

For each question, darken the bubble corresponding to the correct integer in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

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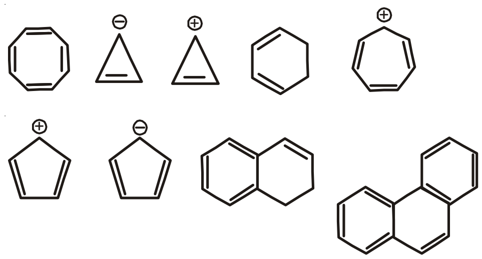
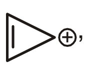
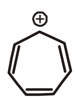
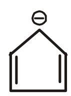
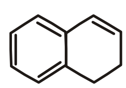
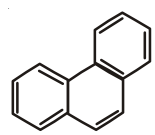
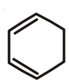
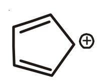
26. Among the following, the number of aromatic compound(s) is



Answer (5)

Sol. , , ,  and  are aromatic.

 and  are non-aromatic while  and  are antiaromatic compounds.

27. Among H_2, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^-, and F_2, the number of diamagnetic species is
(Atomic numbers: H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

Answer (6)

Sol. $H_2\ \sigma_{1s}^2$ Diamagnetic

$He_2^+\ \sigma_{1s}^2\ \sigma_{1s}^{*1}$ Paramagnetic

$Li_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2$ Diamagnetic

$Be_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\sigma_{2s}^{*2}$ Diamagnetic

$B_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\sigma_{2s}^{*2}\ \pi_{2p_x}^1 = \pi_{2p_y}^1$ Paramagnetic

$C_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\sigma_{2s}^{*2}\ \pi_{2p_x}^2 = \pi_{2p_y}^2$ Diamagnetic

$N_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2p_x}^2 = \pi_{2p_y}^2\ \sigma_{2p_z}^2$ Diamagnetic

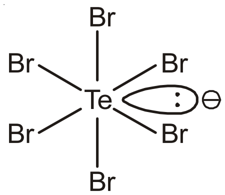
$O_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\sigma_{2s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2\pi_{2p_x}^{*1} = \pi_{2p_y}^{*1}$ Paramagnetic

$F_2\ \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\sigma_{2s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2, \pi_{2p_x}^{*2} = \pi_{2p_y}^{*2}$ Diamagnetic

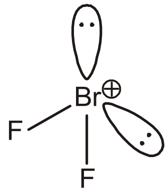
28. The sum of the number of lone pairs of electrons on each central atom in the following species is
$[TeBr_6]^{2-}$, $[BrF_2]^+$, SNF_3 and $[XeF_3]^-$
(Atomic numbers: N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

Answer (6)

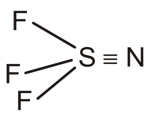
Sol.



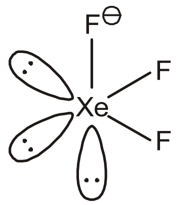
It has one lone pair



It has two lone pairs



It has no lone pair



It has three lone pairs

29. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7}S. The pH of the solution is 4. The value of limiting molar conductivity $\left(\Lambda_m^o\right)$ of this weak monobasic acid in aqueous solution is Z × 10^2 S cm^{-1} mol^{-1}. The value of Z is

Answer (6)

Sol. $\kappa = C\left(\frac{l}{A}\right)$

$$= 5\times10^{-7}\times\frac{120}{1}$$

$$= 6 \times 10^{-5}\ S\ cm^{-1}$$

$$\lambda_M = \frac{\kappa\times1000}{M}$$

$$= \frac{6\times10^{-5}\times1000}{15\times10^{-4}}$$

$\Rightarrow$ λ_M = 40 S cm^2 mol^{-1}.

$[H^+] = C\alpha = 10^{-4}$

$$\alpha = \frac{10^{-4}}{15\times10^{-4}} = \frac{1}{15}$$

$$\Rightarrow \quad \alpha = \frac{\lambda_M}{\lambda_M^o}$$

$$\Rightarrow \quad \lambda_M^\circ = 40 \times 15$$

$$= 600 = 6 \times 10^2\ S\ cm^{-1}\ mol^{-1}$$

30. A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is 8 g cm^{-3}, then the number of atoms present in 256 g of the crystal is N $\times$ 10^{24}. The value of N is

Answer (2)

Sol. $a = 4 \times 10^{-8}$ cm (a = edge length)

$d = 8$ g cm^{-3} (density)

$d = \dfrac{ZM}{N_A a^3}$ M = molecular mass (g/mol)

$Z \rightarrow$ number of atom in 1 unit cell

$$M = \frac{dN_A a^3}{Z} = \frac{8 \times 6 \times 10^{23} \times 64 \times 10^{-24}}{4} = 76.8 \text{ g/mol}$$

Mole of solid in 256 g = 3.33 moles

No. of atom = $3.33 \times N_A = 20 \times 10^{23} = 2 \times 10^{24}$

SECTION - 3 (Maximum Marks : 18)

This section contains **SIX** questions of matching type.

This section contains **TWO** tables (each having 3 columns and 4 rows).

Based on each table, there are **THREE** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

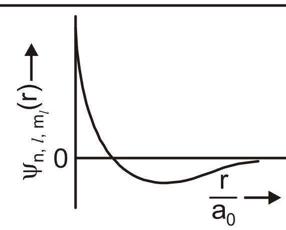
Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks : –1 In all other cases

Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three columns of the following table.

The wave function, ψ_{n,l,m_l} is a mathematical function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, l and m_l. Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the table, Z is atomic number and a_0 is Bohr radius.

Column 1	Column 2	Column 3
(I) 1*s* orbital	(i) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P) 
(II) 2*s* orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_0^3}$
(III) 2p_z orbital	(iii) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} re^{-\left(\frac{Zr}{2a_0}\right)} \cos\theta$	(R) Probability density is maximum at nucleus
(IV) 3d_{z^2} orbital	(iv) xy-plane is a nodal plane	(S) Energy needed to excite electron from n = 2 state to n = 4 state is $\frac{27}{32}$ times the energy needed to excite electron from n = 2 state to n = 6 state

31. For He^+ ion, the only **INCORRECT** combination is

(A) (II) (ii) (Q)
(B) (I) (iii) (R)
(C) (I) (i) (R)
(D) (I) (i) (S)

Answer (B)

Sol. 1*s* orbital is nondirectional so ψ will not depend upon $\cos\theta$.

Hence 'B' is incorrect.

32. For the given orbital in Column 1, the only CORRECT combination for any hydrogen-like species is

(A) (II) (ii) (P)
(B) (III) (iii) (P)
(C) (I) (ii) (S)
(D) (IV) (iv) (R)

Answer (A)

Sol. For H-like species only A is correct because

In (B) $2p_z$ orbital has no radial node

In (C) 1s orbital has no radial node

In (D) for $3d_{z^2}$, xy plane is not nodal plane.

33. For hydrogen atom, the only CORRECT combination is

(A) (I) (iv) (R)
(B) (I) (i) (P)
(C) (I) (i) (S)
(D) (II) (i) (Q)

Answer (C)

Sol. For H-atom

1*s*-orbital $\psi \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$

Also, $E_4 - E_2 = \frac{3}{16}$

$E_6 - E_2 = \frac{2}{9}$

Hence $(E_6 - E_2) \times \frac{27}{32} = E_4 - E_2$

Answer Q.34, Q.35 and Q.36 by appropriately matching the information given in the three columns of the following table.

Columns 1, 2 and 3 contain starting materials, reaction conditions, and type of reactions, respectively.

Column 1	Column 2	Column 3
(I) Toluene	(i) $NaOH/Br_2$	(P) Condensation
(II) Acetophenone	(ii) $Br_2/h\nu$	(Q) Carboxylation
(III) Benzaldehyde	(iii) $(CH_3CO)_2O/CH_3COOK$	(R) Substitution
(IV) Phenol	(iv) $NaOH/CO_2$	(S) Haloform

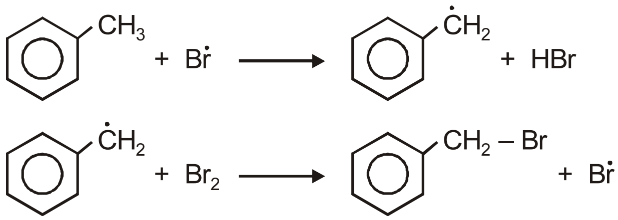
34. The only CORRECT combination in which the reaction proceeds through radical mechanism is

(A) (II) (iii) (R) (B) (III) (ii) (P)

(C) (IV) (i) (Q) (D) (I) (ii) (R)

Answer (D)

Sol. $Br_2 \xrightarrow{h\nu} Br^{\bullet} + Br^{\bullet}$



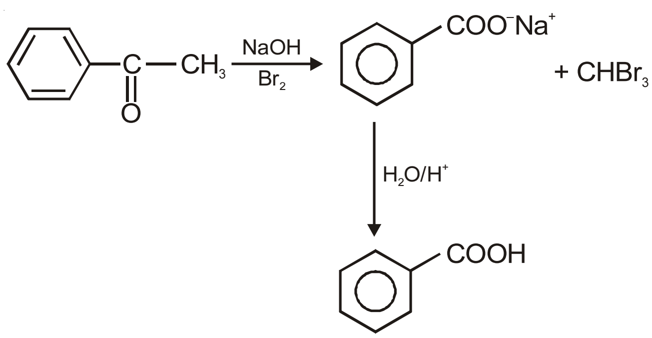
35. For the synthesis of benzoic acid, the only CORRECT combination is

(A) (II) (i) (S) (B) (III) (iv) (R)

(C) (I) (iv) (Q) (D) (IV) (ii) (P)

Answer (A)

Sol.



It is Haloform reaction

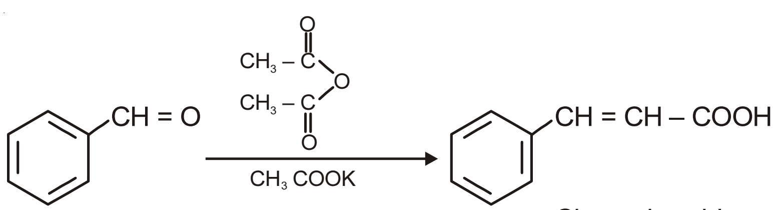
36. The only CORRECT combination that gives two different carboxylic acids is

(A) (I) (i) (S) (B) (II) (iv) (R)

(C) (IV) (iii) (Q) (D) (III) (iii) (P)

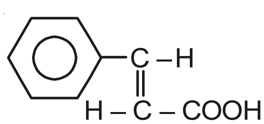
Answer (D)

Sol.

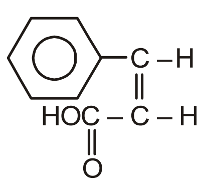


Cinnamic acid

It exists in two geometrical forms.



Trans isomer



Cis isomer

It is basic example of Perkin's reaction.

SECTION - 1 (Maximum Marks : 28)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–2	In all other cases

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

37. Let X and Y be two events such that $P(X) = \frac{1}{3}, P(X \mid Y) = \frac{1}{2}$ and $P(Y \mid X) = \frac{2}{5}$. Then

(A) $P(X \cap Y) = \frac{1}{5}$

(B) $P(X' \mid Y) = \frac{1}{2}$

(C) $P(Y) = \frac{4}{15}$

(D) $P(X \cup Y) = \frac{2}{5}$

Answer (B, C)

Sol. $P(X) = \frac{1}{3}, P\left(\frac{X}{Y}\right) = \frac{1}{2}, P\left(\frac{Y}{X}\right) = \frac{2}{5}$

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$$

$$P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)} = \frac{2}{5}$$

$$\Rightarrow \quad P(Y) = \frac{4}{15}, P(X) = \frac{1}{3}, P(X \cap Y) = \frac{2}{15}$$

$$\therefore \quad P\left(\frac{X'}{Y}\right) = \frac{P(X' \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)} = \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$$

$$P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{9-2}{15} = \frac{7}{15}$$

38. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x?

(A) $1+\sqrt{1+y^2}$

(B) $-1+\sqrt{1-y^2}$

(C) $1-\sqrt{1+y^2}$

(D) $-1-\sqrt{1-y^2}$

Answer (B, D)

Sol. $a - b \neq 1$, $y \neq 0$, $z = x + iy$

$$\text{Im}\left(\frac{az+b}{z+1}\right) = y$$

$$\text{Im}\left(\left(\frac{(ax+b)+ayi}{(x+1)+iy}\right)\times\left(\frac{x+1-iy}{x+1-iy}\right)\right) = y$$

$$\text{Im}\left(\frac{(ax+b)(x+1)+ay^2+ay(x+1)i-iy(ax+b)}{(x+1)^2+y^2}\right) = y$$

$\Rightarrow$ $ay(x + 1) - y(ax + b) = y(x + 1)^2 + y^3$

$\Rightarrow$ $ax + a - ax - b = x^2 + 2x + 1 + y^2$

$\Rightarrow$ $a - b = x^2 + y^2 + 2x + 1$

$\Rightarrow$ $1 = x^2 + y^2 + 2x + 1$

$\Rightarrow$ $(x + 1)^2 = 1 - y^2$

$\Rightarrow$ $x+1 = \pm\sqrt{1-y^2}$

$\Rightarrow$ $x = -1 \pm \sqrt{1-y^2}$

39. Let $f : \mathbb{R} \to (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

(A) $x^9 - f(x)$

(B) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\, dt$

(C) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\, dt$

(D) $e^x - \int_0^x f(t)\sin t\, dt$

Answer (A, C)

Sol. $g(x) = x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\, dt$

$g(0) = 0 - \int_0^{\frac{\pi}{2}} f(t)\cos t\, dt < 0$ as $0 < f(t)\cos t < 1$

$g(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t)\cos t\, dt > 0$

$g(x) = x^9 - f(x) \rightarrow g(0) = 0 - f(0) < 0$

$g(1) = 1 - f(1) > 0$

Also for (0, 1)

$e^x - \int_0^x f(t)\sin t\, dt > 0$

Also, $f(x) + \int_0^{\pi/2} f(t)\sin t\, dt > 0 \quad \forall x \in (0,1)$

40. Let $[x]$ be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous?

(A) $x = 2$

(B) $x = 1$

(C) $x = -1$

(D) $x = 0$

Answer (A, B, C)

Sol. $f(x) = x\cos(\pi(x + [x]))$

$$= \begin{cases} x\cos\pi x, [x] \text{ is even} \\ -x\cos\pi x, [x] \text{ is odd} \end{cases}$$

Clearly $f(1^+) \neq f(1)$, $f(2^+) \neq f(2)$, $f(-1^+) \neq f(-1^-)$

but $f(0) = f(0^+) = f(0) = 0$ hence f is discontinuous at $x = 1, -1, 2$ but continuous at $x = 0$.

41. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k?

(A) $p = -2$, $h = 2$, $k = -4$

(B) $p = 2$, $h = 3$, $k = -4$

(C) $p = -1$, $h = 1$, $k = -3$

(D) $p = 5$, $h = 4$, $k = -3$

Answer (B)

Sol. Equation of chord is $2x + y = p$

For point of intersection with parabola,

$$(p-2x)^2 = 16x$$

$\Rightarrow$ $4x^2 - (4p+16)x + p^2 = 0$

$\Rightarrow$ $p = 128p + 256$

Hence, $p = -2$

Equation of chord for midpoint (h, k)

$$yk - 8(x+h) = k^2 - 16h$$

$\Rightarrow$ $yk - 8x = k^2 - 8h$

$$-8x + ky = k^2 - 8h$$

On comparing,

$$2x + y = p$$

$$-\frac{8}{2} = \frac{k}{1} = \frac{k^2 - 8h}{p}$$

$$k^2 - 8h = -4p$$

$$16 - 8h = -4p$$

$\Rightarrow$ $-p + 2h = 4$

$\therefore$ $k = -4,\ p = 2,\ h = 3$

42. Which of the following is(are) not the square of a 3×3 matrix with real entries?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Answer (B, D)

Sol. Since determinant value of matrix in option (A) & (C) are positive.

Hence it can be represented as square of a matrix, but determinant value of matrix in option (B) & (D) are negative hence, it cannot be represented as square of a matrix.

43. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following cannot be sides of a right angled triangle?

(A) a, 4, 2

(B) a, 4, 1

(C) $2a$, 8, 1

(D) $2a$, 4, 1

Answer (A, B, C)

Sol. $y = 2x + 1$ be tangent to $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$

$\therefore$ $y = 2x + 1$ compare with $y = mx \pm \sqrt{a^2m^2 - 16}$.

$\Rightarrow$ $m = 2$ and $a^2m^2 - 16 = 1$

$\Rightarrow$ $4a^2 = 17$

$\therefore$ $2a$, 4, 1 are the sides of right angled triangle.

SECTION - 2 (Maximum Marks : 15)

This section contains **FIVE** questions.

The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

For each question, darken the bubble corresponding to the correct integer in the ORS.

For each question, marks will be awarded in one of the following categories:

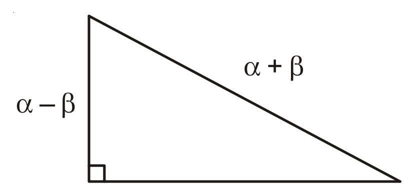
Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

44. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

Answer (6)

Sol. Using $(\alpha + \beta)^2 = \alpha^2 + (\alpha - \beta)^2$



$\Rightarrow$ $\beta = \frac{\alpha}{4}$

Also, $\frac{1}{2}\alpha(\alpha - \beta) = 24$

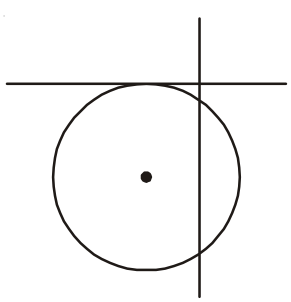
$$\Rightarrow \left.\begin{aligned} \alpha &= 8 \\ \beta &= 2 \end{aligned}\right\}$$

Smallest side = 6

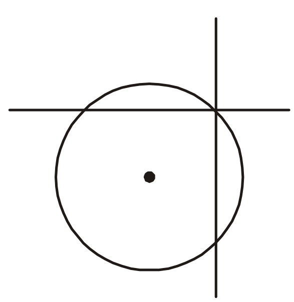
45. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

Answer (2)

Sol. Possible cases



$p = -1$



$p = 0$

46. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right)=3$ and $f'(0) = 1$.

If $g(x)=\int_x^{\frac{\pi}{2}}[f'(t)\operatorname{cosec}t-\cot t\operatorname{cosec}t\, f(t)]dt$ for $x\in\left(0,\frac{\pi}{2}\right]$, then $\lim_{x\to 0}g(x)=$

Answer (2)

Sol. $g(x)=\int_x^{\pi/2} d((\operatorname{cosec}t)(f(t)))=\{(\operatorname{cosec}t).f(t)\}_x^{\pi/2}$

So, $\lim_{x\to 0}g(x)=\lim_{x\to 0}\left(f\left(\frac{\pi}{2}\right)-f(x).\operatorname{cosec}x\right)$

$$=3-\lim_{x\to 0}\frac{f(x)}{\sin x}=3-\lim_{x\to 0}\frac{f'(x)}{\cos x}=2$$

47. For a real number α, if the system $\begin{bmatrix}1&\alpha&\alpha^2\\ \alpha&1&\alpha\\ \alpha^2&\alpha&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Answer (1)

Sol. Equation is rewritten as $AX = B$

where $A=\begin{bmatrix}1&\alpha&\alpha^2\\ \alpha&1&\alpha\\ \alpha^2&\alpha&1\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}1\\-1\\1\end{bmatrix}$

$\Rightarrow\quad X=\frac{\operatorname{adj}(A).B}{\det(A)}$

For infinite solution, $\det(A) = 0$ and $\operatorname{adj}(A).B = 0$

$\Rightarrow\quad \alpha = 1, -1$

but α is not equal to 1 as in this case equation is inconsistent.

So, $\alpha = -1$

i.e., $\boxed{1+\alpha+\alpha^2=1}$

48. Words of length 10 are formed using the letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J*. Let *x* be the number of such words where no letter is repeated; and let *y* be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}=$

Answer (5)

Sol. Clearly $x = 10!$

For calculating *y*, one letter has to be excluded, this can be done in 10 ways.

Out of the remaining 9 letters, one of them is to be repeated

$$y={}^{10}C_1\times{}^{9}C_1\times\frac{10!}{2!}=5\times 9\times 10!$$

$$\therefore\quad \frac{y}{9x}=\frac{5\times 9\times 10!}{9\times 10!}=5$$

SECTION - 3 (Maximum Marks : 18)

This section contains **SIX** questions of matching type.

This section contains **TWO** tables (each having 3 columns and 4 rows).

Based on each table, there are **THREE** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–1	In all other cases

Answer Q.49, Q.50 and Q.51 by appropriately matching the information given in the three columns of the following table.

Column 1, 2, and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4,$ then which of the following options is the only correct combination?

(A) (II) (iv) (R)

(B) (IV) (iii) (S)

(C) (II) (iii) (R)

(D) (IV) (iv) (S)

Answer (A)

50. For $a = \sqrt{2},$ if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only correct combination for obtaining its equation?

(A) (I) (i) (P)

(B) (III) (i) (P)

(C) (II) (ii) (Q)

(D) (I) (ii) (Q)

Answer (D)

51. If a tangent to a suitable conic (column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only correct combination?

(A) (III) (i) (P)   (B) (I) (ii) (Q)

(C) (II) (iv) (R)   (D) (III) (ii) (Q)

Answer (A)

Solution of Q. Nos. 49 to 51

Correct combinations are

I, ii, Q

II, iv, R

III, i, P

IV, iii, S

49. Out of the given *A*, *B* are the right combination which we need to check. Out of this (Less check for)

$x^2 + a^2y^2 = a^2$,

$$\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right) = \left(\frac{+4\times\frac{\sqrt{3}}{2}}{2}, \frac{1}{2}\right) = \left(\frac{1}{2}\right)$$

Which is given so, (A) is correct.

50. (–1, 1) for $a = \sqrt{2}$ lies on

$x^2 + y^2 = a^2$, so the only correct answer is (D).

51. As $y = x + 8$ and its point of contact (8, 16) satisfy only $y^2 = 4ax$

Answer Q.52, Q.53 and Q.54 by appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x\log_e x, x \in (0, \infty)$

- Column 1 contains information about zeros of $f(x), f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behaviour of $f(x), f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim\limits_{x\to\infty} f(x) = 0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim\limits_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim\limits_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim\limits_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

52. Which of the following options is the only correct combination?

(A) (II) (ii) (Q)   (B) (III) (iii) (R)

(C) (IV) (iv) (S)   (D) (I) (i) (P)

Answer (A)

53. Which of the following options is the only correct combination?

(A) (III) (iv) (P) (B) (II) (iii) (S)

(C) (I) (ii) (R) (D) (IV) (i) (S)

Answer (B)

54. Which of the following options is the only incorrect combination?

(A) (I) (iii) (P) (B) (II) (iii) (P)

(C) (III) (i) (R) (D) (II) (iv) (Q)

Answer (C)

Solution of Q. Nos. 52 to 54

$$f(x) = x + \log_e x - x\log_e x$$

(I) $f(1) = 1 + \log 1 - 1\log 1 = 1 > 0$

$$f(e^2) = e^2 + \log_e e^2 - e^2 \log_e e^2 = e^2 + 2 - 2e^2 = 2 - e^2 < 0$$

$\Rightarrow$ (I) is true.

(II) $f'(x) = 1 + \dfrac{1}{x} - \log_e x - 1$

$f'(1) = 1$

$f'(e) = \dfrac{1}{e} - 1 < 0$

$\Rightarrow$ (II) is true.

(III) $f''(x) = -\dfrac{1}{x^2} - \dfrac{1}{x} < 0$ for all $x \in (0, \infty)$

So, $f'(x)$ is decreasing

So, min. of $f'(x)$ in (0, 1) is $f'(1) = 1$

So, (III) is false.

(IV) $f''(1) = -2 < 0$

$f''(e) = -\dfrac{1}{e^2} - \dfrac{1}{e} < 0$

So, (IV) is false.

Column 2

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x + \log_e x - x\log_e x = \lim_{t\to 0^+} \frac{1}{t} - \log_e t + \frac{\log_e t}{t}$$

$$\lim_{t\to 0^+} \frac{1 - t\log_e t + \log_e t \to -\infty}{t \to 0^+} = -\infty$$

(i) is false

(ii) is true

(iii) $\lim\limits_{x\to\infty} f'(x) = \lim\limits_{x\to\infty} \frac{1}{x} - \log_e x = \lim\limits_{t\to 0^+} t + \log_e t = -\infty$ (True)

(iv) $\lim\limits_{x\to\infty} f''(x) = -\frac{1}{x^2} - \frac{1}{x} = \lim\limits_{x\to\infty} \frac{-1-x}{x^2} = 0$ (True)

Column 3

(P) $f'(x) = \frac{1}{x} - \log_e x > 0$ is true

for $x \in (0,\ 1),\ \frac{1}{x} \in (1,\ \infty)$

$\log_e x \in (-\infty,\ 0)$

(Q) $f'(x) = \frac{1}{x} - \log_e x$ is decreasing function

$f'(e) = \frac{1}{e} - 1 < 0$

$f'(e^2) = \frac{1}{e^2} - 2 < 0$ is true

(R) $f''(x) = -\frac{1}{x} - \frac{1}{x^2} < 0$ so, $f'(x)$ is decreasing

So, (R) is false.

(S) is true.