JEE ADVANCED PAPER-II

Time Duration: 3 Hours **Maximum Marks : 183**

INSTURCTIONS:

QUESTION PAPER FORMAT AND MARKING SCHEME :

1. The question paper has **three parts** : Physics, Chemistry and Mathematics.

2. Each part has three sections as detailed in the following table :

Section	Question Type	Number of Questions	Category-wise Marks for Each Question				Maximum Marks of the Section
			Full Marks	Partial Marks	Zero Marks	Negative Marks	
1	Single Correct Option	7	+3 If only the bubble corresponding to the correct option is darkened	—	0 If none of the bubbles is darkened	–1 In all other cases	21
2	One or more correct option(s)	7	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	–2 In all other cases	28
3	Compre-hension	4	+3 If only the bubble corresponding to the correct answer is darkened	—	0 In all other cases	—	12

PHYSICS

SECTION - 1 (Maximum Marks : 21)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–1	In all other cases

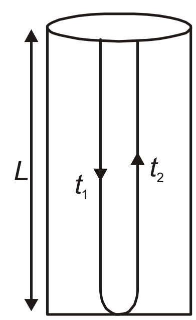
1. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10\ \text{ms}^{-2}$ and the velocity of sound is $300\ \text{ms}^{-1}$. Then the fractional error in the measurement, $\delta L/L$, is closest to

(A) 5% (B) 3%

(C) 1% (D) 0.2%

Answer (C)

Sol. $t_1 = \sqrt{\frac{2L}{g}}$



$t_2 = \frac{L}{V}$

$\therefore\quad T = t_1 + t_2$

$\Rightarrow\quad T = \sqrt{\frac{2L}{g}} + \frac{L}{V}$

$\Rightarrow\quad \Delta T = \sqrt{\frac{2}{g}} \times \frac{1}{2\sqrt{L}}\Delta L + \frac{1}{V}\Delta$

$\Rightarrow\quad 0.01 = \left(\frac{1}{\sqrt{5}} \times \frac{1}{2 \times \sqrt{20}} + \frac{}{300}\right)\Delta L$

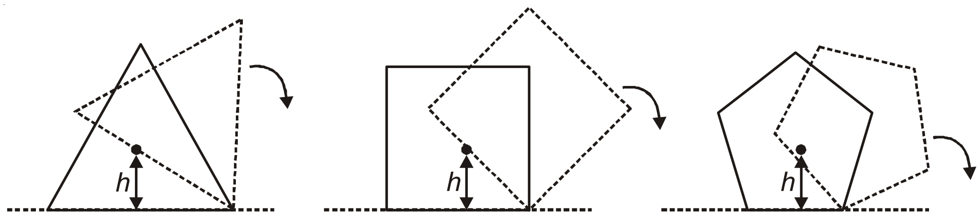
$\Rightarrow\quad 0.01 = \left(\frac{1}{20} + \frac{1}{300}\right)\Delta L$

$\Rightarrow\quad 0.01 = \frac{(15+1)}{300}\Delta L$

$\Rightarrow\quad \Delta L = \frac{0.01 \times 300}{16}$

$\therefore\quad \frac{\Delta L}{L} \times 100 = \frac{3}{16 \times 20} \times 100 = 1\%$

2. Consider regular polygons with number of sides n = 3, 4, 5.... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

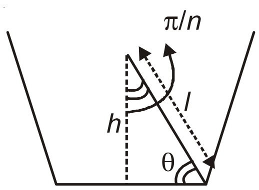
(B) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

(C) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

(D) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

Answer (C)

Sol.



$$\theta = \frac{\pi}{2} - \frac{\pi}{n}$$

$$\sin\theta = \cos\frac{\pi}{n}$$

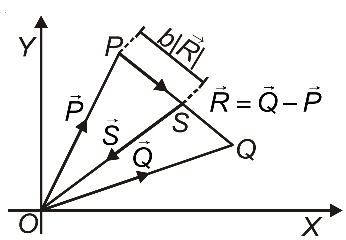
$$l = \frac{h}{\sin\theta}$$

$$\Delta = l - h$$

$$= h\left[\frac{1}{\sin\theta} - 1\right]$$

$$= h\left[\frac{1}{\cos\frac{\pi}{n}} - 1\right]$$

3. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is



(A) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(B) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

(C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(D) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

Answer (C)

Sol. $\vec{S} = \vec{P} + b|\vec{R}|\hat{R}$

$$= \vec{P} + b|\vec{R}|\frac{\vec{R}}{|\vec{R}|}$$

$$= \vec{P} + b\vec{R}$$

$$= \vec{P} + b\left(\vec{Q} - \vec{P}\right)$$

$$= (1-b)\vec{P} + b\vec{Q}$$

4. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to

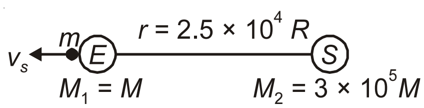
(Ignore the rotation and revolution of the Earth and the presence of any other planet)

(A) $v_s = 22$ km s^{-1} (B) $v_s = 42$ km s^{-1}

(C) $v_s = 62$ km s^{-1} (D) $v_s = 72$ km s^{-1}

Answer (B)

Sol.



loss in KE = Gain in PE

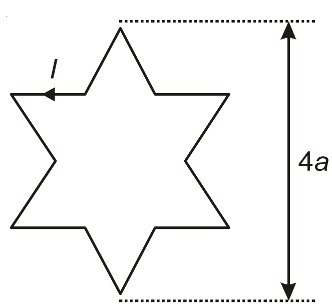
$$\Rightarrow \frac{1}{2}mv_s^2 = \frac{GM_1m}{R} + \frac{GM_2m}{r}$$

$$\Rightarrow \frac{1}{2}v_s^2 = \frac{GM}{R} + \frac{G \times 3 \times 10^5 M}{2.5 \times 10^4 R}$$

$$\Rightarrow v_s = \sqrt{2 \times \frac{GM}{R} \times 13} = 11.2 \times \sqrt{13} = 40.4 \text{ km/s}$$

$$\simeq 42 \text{ km/s}$$

5. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the center of the loop is



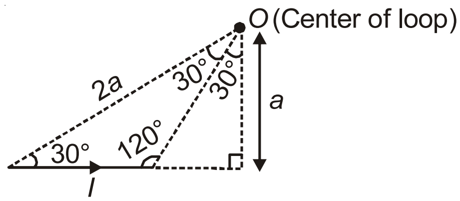
(A) $\frac{\mu_0 I}{4\pi a}3\left[2-\sqrt{3}\right]$ (B) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}-1\right]$

(C) $\frac{\mu_0 I}{4\pi a}3\left[\sqrt{3}-1\right]$ (D) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}+1\right]$

Answer (B)

Sol. Considering one section out of symmetric star shaped conducting wire loop.

From geometry :



Magnetic field at the center of the loop due to all 12 identical sections is additive in nature.

$$\therefore \quad B_{net} = 12 \times \frac{\mu_0 I}{4\pi a}[\cos 30^\circ + \cos 120^\circ]$$

$$= \frac{\mu_0 I}{4\pi a} \cdot 6 \cdot \left[\sqrt{3} - 1\right]$$

6. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda \left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ result in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(A) λ_d^3 / λ

(B) λ_d^2 / λ^2

(C) λ_d^3 / λ^2

(D) λ_d / λ

Answer (C)

Sol. $\lambda_d = \dfrac{h}{\sqrt{\left(\frac{hc}{\lambda} - \phi\right)2m}}$

$$\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)} = \frac{h}{\lambda_d}$$

$$2m\left(\frac{hc}{\lambda} - f\right) = \frac{h^2}{\lambda_d^2}$$

$$2m\ hc\left(\frac{-1}{\lambda^2}d\lambda\right) = -\frac{h^2}{\lambda_d^3}$$

$$\frac{d\lambda_d}{d\lambda} = k\frac{\lambda_d^3}{\lambda^2}$$

7. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

(A) $R^{2/3}$

(B) $\frac{1}{R}$

(C) R

(D) R^3

Answer (C)

Sol. $M = \frac{4}{3}\pi R^3 \rho$

$$0 = \frac{4}{3}\pi\left[3R^2 \frac{dR}{dt}\rho + R^3 \frac{d\rho}{dt}\right]$$

Dividing by ρ

$$0 = 3R^2 \frac{dR}{dt} + R^3 \frac{1}{\rho} \cdot \frac{d\rho}{dt}$$

$$\frac{dR}{dt} 3R^2 = -R^3 K$$

$$\frac{dR}{dt} \propto R$$

SECTION - 2 (Maximum Marks : 28)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–2	In all other cases

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

8. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_x = V_0 \sin \omega t,$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(A) $V_{XY}^{rms} = V_0$

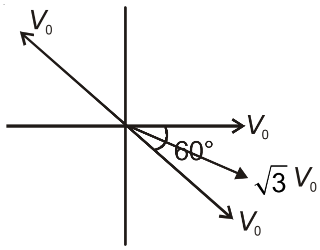
(B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(D) Independent of the choice of the two terminals

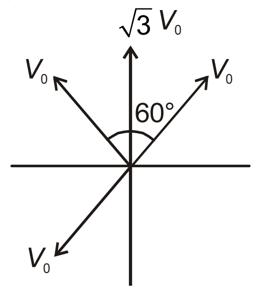
Answer (C, D)

Sol. $V_{XY} = V_0 \sin \omega t - V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$



$V_{XY}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}}$

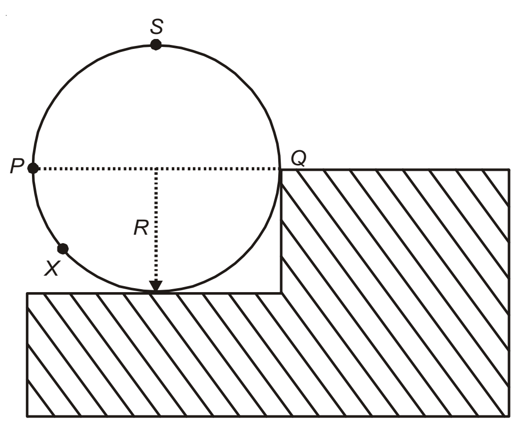
$V_{YZ} = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$



$V_{YZ}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}}$

$\therefore$ C, D

9. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



(A) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

(B) If the force is applied normal to the circumference at point P then τ is zero

(C) If the force is applied normal to the circumference at point X then τ is constant

(D) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

Answer (B, C)

Sol. Correct options (B, C)   [Treating magnitude of force constant]

For option (B):

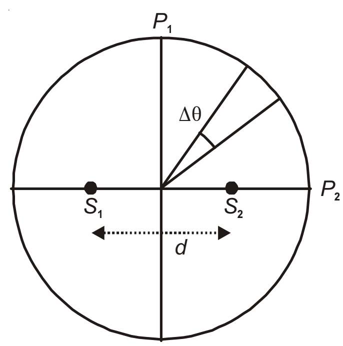
Applied force passes through point *Q*.

So, its torque is zero.

For option (C):

Torque due to applied force at *x* remains constant.

10. Two coherent monochromatic point sources S_1 and S_2 of wavelength λ = 600 nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance *d* = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



(A) A dark spot will be formed at the point P_2

(B) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

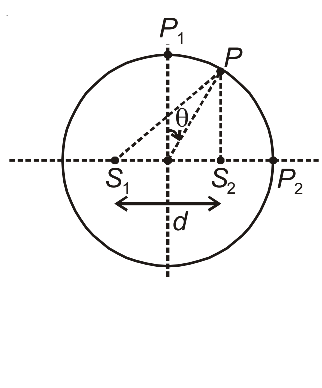
(C) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

(D) At P_2 the order of the fringe will be maximum

Answer (C, D)

Sol. $d = 1.8 \times 10^{-3}$ m

$= 18 \times 10^{-4}$ m



and $\lambda = 6 \times 10^{-7}$ m

Path difference at point *P* (as shown)

$\Delta x = S_1P - S_2P = d \sin\theta$, where θ angle is measured from vertical line as shown.

For bright fringe $d \sin\theta = m\lambda$ ...(i)

Point P_1 is the point of central maxima.

At point P_2, path difference $(\Delta x) = d$

If P_2 is the point of bright fringe, then

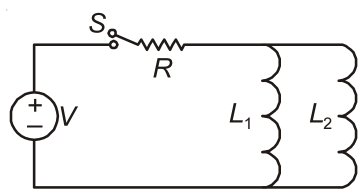
$$d = m\lambda \Rightarrow m = \frac{d}{\lambda} = 3000$$

On differentiating equation (i)

$d \cos\theta\, (\Delta\theta) = (\Delta m)\, \lambda$ = constant for consecutive bright fringe

$\cos\theta \downarrow \therefore \Delta\theta \uparrow$ as θ varies from 0 to $\frac{\pi}{2}$

11. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



(A) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(B) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

(C) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)

(D) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

Answer (A, C, D)

Sol. Final current through battery = $\frac{V}{R}$

$\therefore$ Current through L_1 = $\frac{V}{R}\left(\frac{L_2}{L_1+L_2}\right)$

Current through L_2 = $\frac{V}{R}\left(\frac{L_1}{L_1+L_2}\right)$

At $t = 0$ current through source = zero

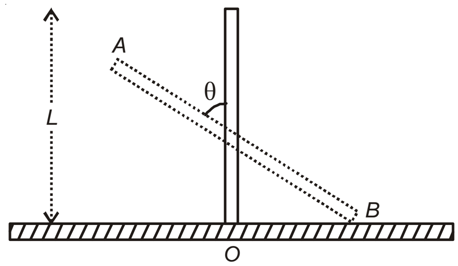
At any time i = $\frac{V}{R} + \left(0 - \frac{V}{R}\right)e^{\frac{-tR}{\left(\frac{L_1L_2}{L_1+L_2}\right)}}$

$\therefore$ Current through L_1 = $i\frac{L_2}{L_1+L_2} = i_1$

Current through L_2 = $\frac{iL_1}{L_1+L_2} = i_2$

$$\frac{i_1}{i_2} = \frac{L_2}{L_1}$$

12. A rigid uniform bar *AB* of length *L* is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



(A) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(B) The midpoint of the bar will fall vertically downward

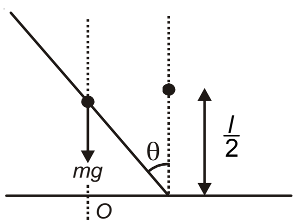
(C) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

(D) The trajectory of the point *A* is a parabola

Answer (A, B, C)

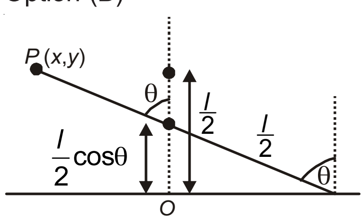
Sol. Torque about *O*, at any instant is $mg.\frac{l}{2}\sin\theta$.

$\therefore$ Option (A)



As no external force acts along *x*-axis, therefore centre of mass will fall vertically downward.

$\therefore$ Option (B)



Displacement of centre of mass along *y*-axis

$$= \frac{l}{2}[1 - \cos\theta]$$

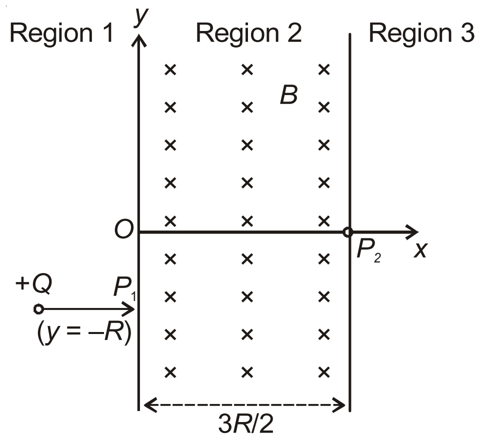
$\therefore$ Option (C)

$$x = -\frac{l}{2}\sin\theta,\ y = l\cos\theta$$

$$\Rightarrow \left(-\frac{2x}{l}\right)^2 + \frac{y^2}{l^2} = 1$$

$\Rightarrow$ Trajectory is not parabola

13. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along x-axis enters region 2 from region 1 at point P_1 ($y = -R$). Which of the following option(s) is/are correct?



(A) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $p/\sqrt{2}$

(B) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

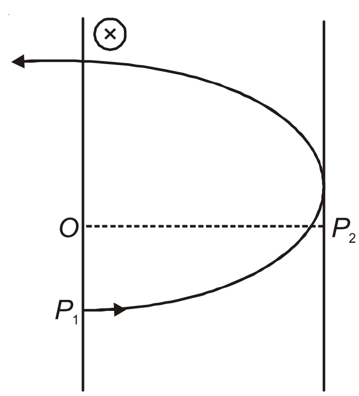
(C) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

(D) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

Answer (B, D)

Sol. The particle will follow circular trajectory inside the magnetic field region. The magnetic field cannot change the magnitude of velocity and momentum.

For longest possible path, the radius of circular motion can be $\frac{3R}{2}$.



At farthest point from y-axis, the momentum is directed upwards.

$\therefore |\Delta\vec{p}| = \sqrt{2}p$

The radius and hence separation between p_1 and re-entry point is proportional to m, if Q, v, B are same.

The particle will return to region only if it completes the half circle.

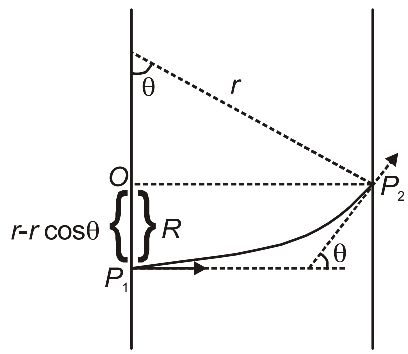
$$r \le \frac{3R}{2}$$

$$\frac{mV}{B} \le \frac{3R}{2}$$

$$\frac{p}{QB} \le \frac{3R}{2}$$

$$B \ge \frac{2p}{3QR}$$

If $B = \dfrac{8p}{13QR}; r = \dfrac{p}{QB} = \dfrac{13R}{8}$



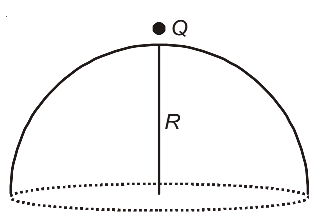
It passes through point P_2 if $r - r\cos\theta = R$

$$\sin\theta = \frac{3\frac{R}{2}}{r} = \frac{12}{13}$$

$$\frac{13R}{8}\left(1 - \frac{5}{13}\right) = R$$

$R = R$

14. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



(A) The component of the electric field normal to the flat surface is constant over the surface

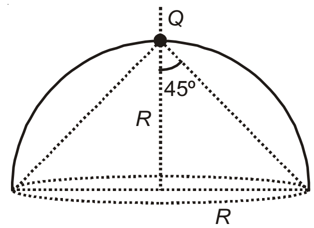
(B) Total flux through the curved and the flat surfaces is $\dfrac{Q}{\varepsilon_0}$

(C) The electric flux passing through the *curved* surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1 - \dfrac{1}{\sqrt{2}}\right)$

(D) The circumference of the flat surface is an equipotential

Answer (C, D)

Sol. Net flux through curved surface and flat surface = 0



$\Rightarrow \quad \phi_{Curved} = -\phi_{Plane}$

$$= -\left[\frac{Q}{2\varepsilon_0}(1-\cos\theta)\right] = -\left[\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

The circumference points are equidistant from Q

$\therefore$ All points will be at the same potential.

$\therefore$ Option (C) and (D) are correct.

SECTION - 3 (Maximum Marks : 12)

This section contains **TWO** Paragraphs.

Based on each paragraph, there are **TWO** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble(s) corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 In all other cases

PARAGRAPH 1

Consider a simple *RC* circuit as shown in Figure 1.

Process 1: In the circuit the switch *S* is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time $T >> RC$). In the process some dissipation (E_D) occurs across the resistance *R*. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

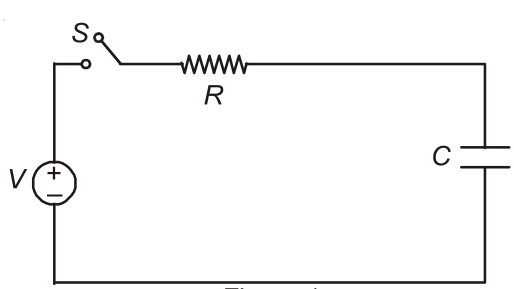


Figure 1

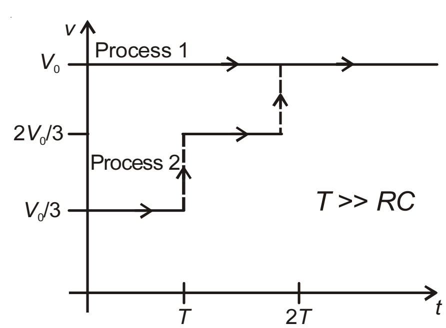


Figure 2

15. In Process 2, total energy dissipated across the resistance E_D is :

(A) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(B) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

(C) $E_D = 3CV_0^2$

(D) $E_D = \frac{1}{2}CV_0^2$

Answer (B)

Sol. $E_D = W_b - \Delta V$

$$= \frac{CV_0}{3}\left[\frac{V_0}{3} + \frac{2V_0}{3} + V_0\right] - \frac{1}{2}CV_0^2$$

$$= \frac{CV_0}{3}\left[\frac{V_0 + 2V_0 + 3V_0}{3}\right] - \frac{1}{2}CV_0^2$$

$$= \frac{CV_0}{3}[2V_0] - \frac{1}{2}CV_0^2$$

$$= \left(\frac{2}{3} - \frac{1}{2}\right)CV_0^2$$

$$= \frac{CV_0^2}{6}$$

16. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by :

(A) $E_C = 2E_D$

(B) $E_C = \frac{1}{2}E_D$

(C) $E_C = E_D$

(D) $E_C = E_D$ In 2

Answer (C)

Sol. Final charge on capacitor = CV

$$W_b = CV^2$$

$$E_c = \frac{1}{2}CV^2$$

$$E_D = W_b - \Delta E_c$$

$$= CV^2 - \frac{1}{2}CV^2$$

$$= \frac{1}{2}CV^2$$

$$\boxed{E_C = E_D}$$

PARAGRAPH 2

One twirls a circular ring (of mass *M* and radius *R*) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is *r*. The finger rotates with an angular velocity ω_0. The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is *g*.

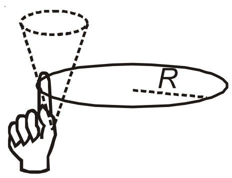


Figure 1

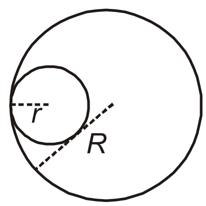


Figure 2

17. The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\dfrac{g}{\mu(R-r)}}$

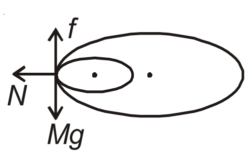
(B) $\sqrt{\dfrac{g}{2\mu(R-r)}}$

(C) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$

(D) $\sqrt{\dfrac{2g}{\mu(R-r)}}$

Answer (A)

Sol.



$N = M\omega^2 (R-r)$

$f = Mg$

$f \le \mu N$

$Mg \le \mu M\omega^2 (R-r)$

$\omega_0 = \sqrt{\dfrac{g}{\mu(R-r)}}$

18. The total kinetic energy of the ring is

(A) $\dfrac{3}{2}M\omega_0^2(R-r)^2$

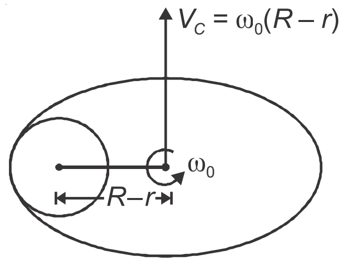
(B) $\dfrac{1}{2}M\omega_0^2(R-r)^2$

(C) $M\omega_0^2R^2$

(D) $M\omega_0^2(R-r)^2$

Answer (C)

Sol. $k = \frac{1}{2}Mv_c^2 + \frac{1}{2}I_c\omega^2$



$$= \frac{1}{2}M\omega_0^2(R-r)^2 + \frac{1}{2}MR^2\omega_0^2$$

$$= \frac{1}{2}M\omega_0^2R^2\left(1-\frac{r}{R}\right)^2 + \frac{1}{2}M\omega_0^2R^2$$

$r << R$

$\frac{r}{R} \rightarrow 0$

$$k = \frac{1}{2}M\omega_0^2R^2 + \frac{1}{2}M\omega_0^2R^2$$

$$= M\omega_0^2R^2$$

CHEMISTRY

SECTION - 1 (Maximum Marks : 21)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks : –1 In all other cases

19. For the following cell,

$Zn(s) \mid ZnSO_4(aq) \parallel CuSO_4(aq) \mid Cu(s)$

when the concentration of Zn^{2+} is 10 times the concentration of Cu^{2+}, the expression for ΔG (in J mol^{-1}) is

[F is Faraday constant; R is gas constant; T is temperature; E°(cell) = 1.1 V]

(A) –2.2F (B) 2.303RT – 2.2F

(C) 1.1F (D) 2.303RT + 1.1F

Answer (B)

Sol. $Zn \mid \underset{(aq)}{ZnSO_4} \parallel \underset{(aq)}{CuSO_4} \mid Cu$

$\Delta G = \Delta G° + RT \ln Q$

$\Delta G = \Delta G° + 2.303\ RT \log Q$ $\left(Q = \frac{[Zn^{+2}]}{[Cu^{+2}]} = \frac{10}{1}\right)$

$\Delta G° = -\ nF\ E°_{Cell}$

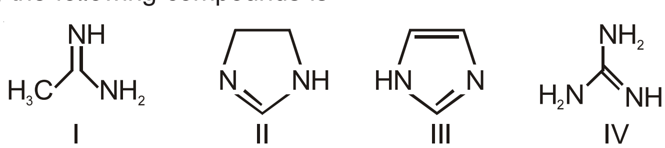
$= -\ 2F \times 1.1$

$= -\ 2.2\ F$

$\Delta G = -\ 2.2\ F + 2.303\ RT \log \frac{10}{1}$

$\boxed{\Delta G = 2.303\ RT - 2.2\ F}$

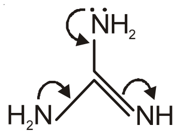
20. The order of basicity among the following compounds is



(A) IV > II > III > I (B) I > IV > III > II

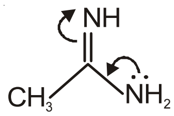
(C) II > I > IV > III (D) IV > I > II > III

Answer (D)

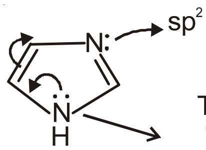
Sol. 

IV

Resonance with two $-\ddot{N}H_2$ groups increases electron density on 'N' of $=NH$

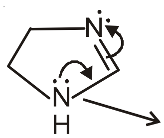


Lesser increase of electron density on = NH due to only one resonance with one $-NH_2$



This LPe^- is not available as it is involve in aromatic Sextet.
'N' is bonded to *sp*^{2} C on both sides.

III



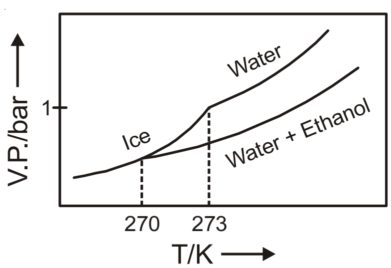
This LPe^- is not involve in aromaticity. So more available
Also, 'N' is bonded to sp^3 C on one side.

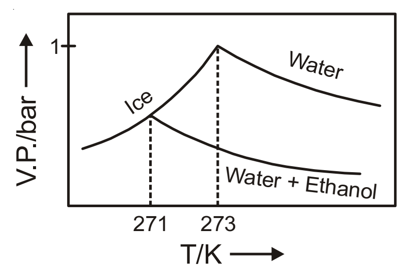
II

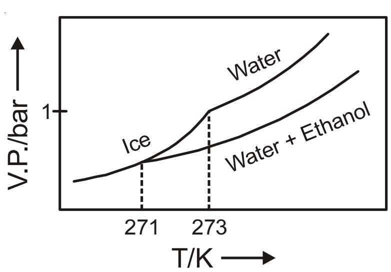
$\therefore$ IV > I > II > III

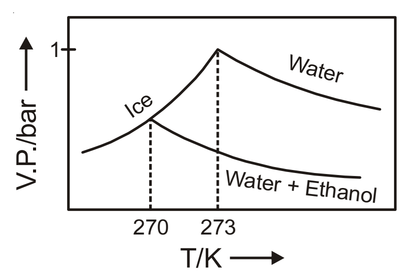
21. Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol^{-1}. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is 46 g mol^{-1}]

Among the following, the option representing change in the freezing point is

(A) 

(B) 

(C) 

(D) 

Answer (A)

Sol. $\Delta T_f = iK_f\left[\dfrac{W_2 \times 1000}{M_2 \times W_1}\right]$

$= 1 \times 2\left[\dfrac{34.5 \times 1000}{46 \times 500}\right]$

$= 3$ K

$273\ (K) - T_f = 3\ (K)$

$\Rightarrow\ T_f = 270\ K$

Also, with decrease in temperature, V.P. decreases.

$\therefore$ Graph (A) is correct.

22. The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at T = 298 K are

$\Delta_f G°[C(graphite)] = 0\ kJ\ mol^{-1}$

$\Delta_f G°[C(diamond)] = 2.9\ kJ\ mol^{-1}$

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by $2 \times 10^{-6}\ m^3\ mol^{-1}$. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information : $1\ J = 1\ kg\ m^2 s^{-2}$; $1\ Pa = 1\ kg\ m^{-1}\ s^{-2}$; $1\ bar = 10^5\ Pa$]

(A) 58001 bar
(B) 1450 bar
(C) 14501 bar
(D) 29001 bar

Answer (C)

Sol. $\Delta G° = \Delta V \cdot \Delta P$

$\Rightarrow 2900 = 2 \times 10^{-6}\ \Delta P$

$\Rightarrow \Delta P = \frac{2900 \times 10^6}{2} Pa$

$P_F - 1 = 14500$ bar

$\Rightarrow P_F = 14501$bar

23. The order of the oxidation state of the phosphorus atom in H_3PO_2, H_3PO_4, H_3PO_3 and $H_4P_2O_6$ is

(A) $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$
(B) $H_3PO_4 > H_3PO_2 > H_3PO_3 > H_4P_2O_6$
(C) $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$
(D) $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$

Answer (C)

Sol. Oxidation state

H_3PO_4 P = + 5

$H_4P_2O_6$ P = + 4

H_3PO_3 P = + 3

H_3PO_2 P = + 1

$H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

24. Which of the following combination will produce H_2 gas?

(A) Cu metal and conc. HNO_3
(B) Au metal and NaCN(aq) in the presence of air
(C) Zn metal and NaOH(aq)
(D) Fe metal and conc. HNO_3

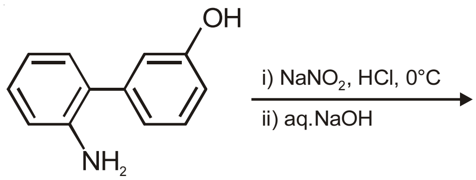
Answer (C)

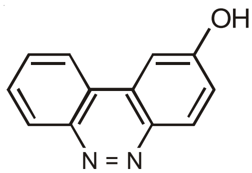
Sol. $Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

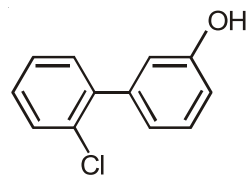
Iron become passive with conc. HNO_3.

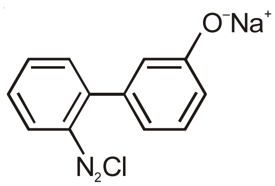
Copper liberate NO_2 with HNO_3

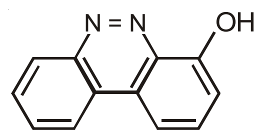
25. The major product of the following reaction is



(A) 

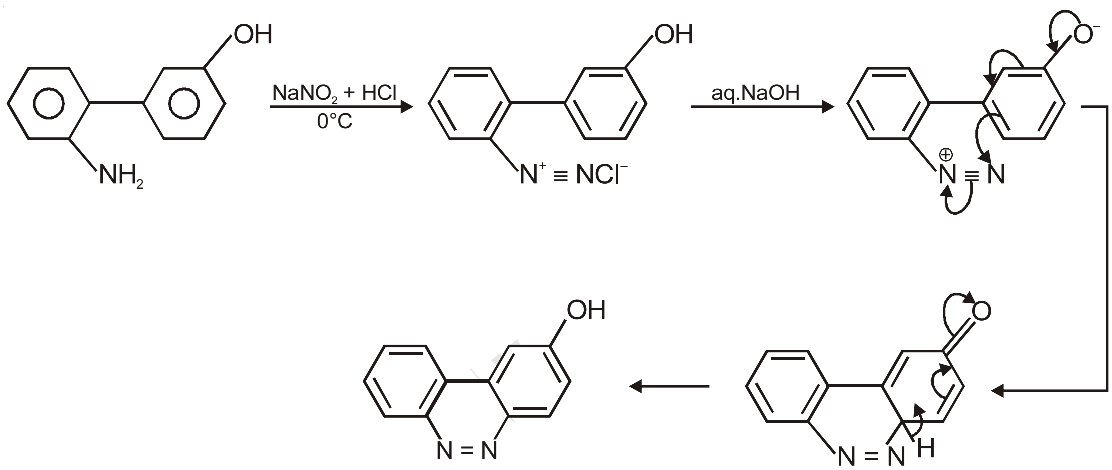
(B) 

(C) 

(D) 

Answer (A)

Sol.



SECTION - 2 (Maximum Marks : 28)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–2	In all other cases

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

26. In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are)

(A) The activation energy of the reaction is unaffected by the value of the steric factor

(B) The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

(C) Since P = 4.5, the reaction will not proceed unless an effective catalyst is used

(D) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

Answer (A, D)

Sol. Steric factor $= \frac{A_{experimental}}{A_{calculated}}$

Steric factor = 4.5

It means $A_{experimental} > A_{calculated}$

[This seems that reaction occurs more quickly than particles collide, thus concept of steric factor was introduced]

27. For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by

(A) With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive

(B) With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative

(C) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases

(D) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

Answer (A, C, D)

Sol. Whether reaction is endothermic or exothermic in forward direction increase in temperature cause intake of heat from surrounding to system in endothermic direction due to which entropy change in system is positive and ΔS of surrounding is negative.

28. The option(s) with only amphoteric oxides is(are)

(A) Cr_2O_3, CrO, SnO, PbO

(B) Cr_2O_3, BeO, SnO, SnO_2

(C) NO, B_2O_3, PbO, SnO_2

(D) ZnO, Al_2O_3, PbO, PbO_2

Answer (B, D)

Sol. ZnO, Al_2O_3, PbO, PbO_2, Cr_2O_3, BeO, SnO and SnO_2 are amphoteric oxides.

NO is neutral oxide

CrO is basic oxide

B_2O_3 is acidic oxide

29. Among the following, the correct statement(s) is(are)

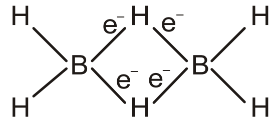
(A) BH_3 has the three-centre two-electron bonds in its dimeric structure

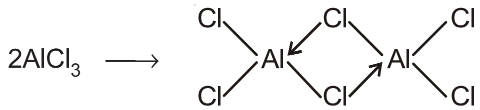
(B) The Lewis acidity of BCl_3 is greater than that of $AlCl_3$

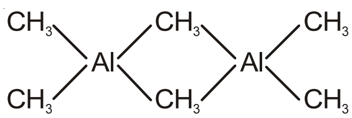
(C) $Al(CH_3)_3$ has the three-centre two-electron bonds in its dimeric structure

(D) $AlCl_3$ has the three-centre two-electron bonds in its dimeric structure

Answer (A, B, C)

Sol.  It has two 3c-2e bonds.

 No 3c-2e bond

 Has two 3c-2e bonds.

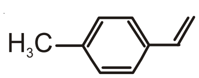
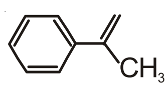
Also BCl_3 is stronger lewis acid than $AlCl_3$.

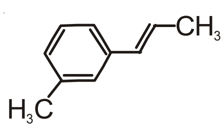
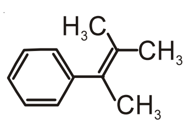
30. Compounds **P** and **R** upon ozonolysis produce **Q** and **S**, respectively. The molecular formula of **Q** and **S** is C_8H_8O. **Q** undergoes Cannizzaro reaction but not haloform reaction, whereas **S** undergoes haloform reaction but not Cannizzaro reaction

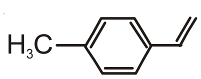
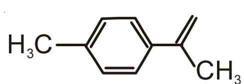
(i) $\mathbf{P} \xrightarrow[\text{ii) Zn/H}_2\text{O}]{\text{i) O}_3\text{/CH}_2\text{Cl}_2} \underset{(C_8H_8O)}{\mathbf{Q}}$

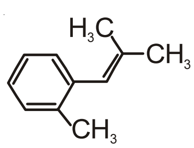
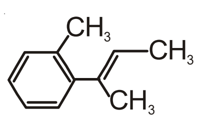
(ii) $\mathbf{R} \xrightarrow[\text{ii) Zn/H}_2\text{O}]{\text{i) O}_3\text{/CH}_2\text{Cl}_2} \underset{(C_8H_8O)}{\mathbf{S}}$

The option(s) with suitable combination of **P** and **R**, respectively, is(are)

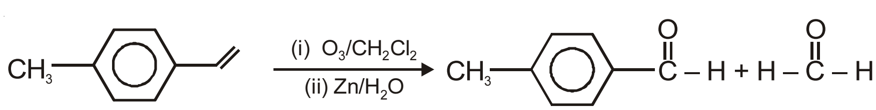
(A)  and 

(B)  and 

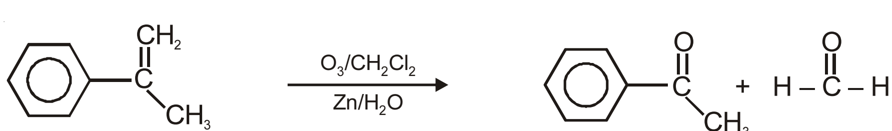
(C)  and 

(D)  and 

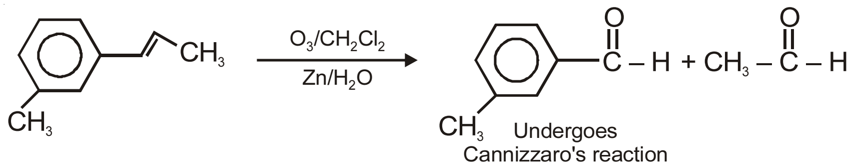
Answer (A, B)

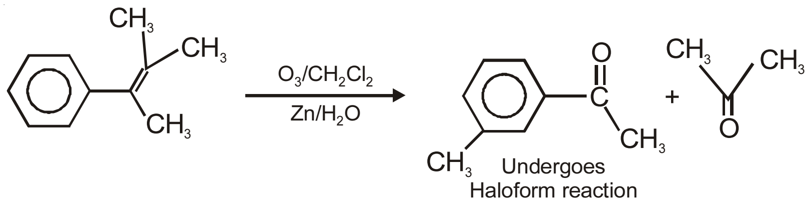
Sol. 

Undergoes Cannizzaro's reaction



Undergoes Haloform reaction





31. The correct statement(s) about surface properties is(are)

(A) Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution

(B) The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature

(C) Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium

(D) Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system

Answer (B, D)

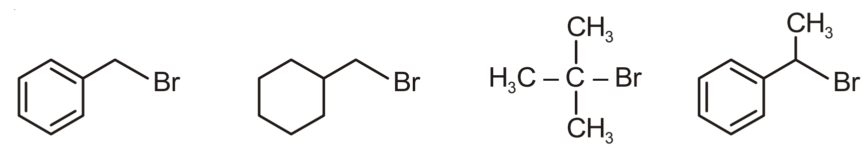
Sol. Adsorption is an exothermic process and is accompanied by decrease is entropy,

$\Delta H_{sys} < 0, \ \Delta S_{sys} < 0$

More is critical temperature (T_c), more are intermolecular forces of attraction.

∴ More is extent of adsorption.

32. For the following compounds, the correct statement(s) with respect to nucleophilic substitution reaction is(are)



I **II** **III** **IV**

(A) **I** and **II** follow S_N2 mechanism

(B) **I** and **III** follow S_N1 mechanism

(C) Compound **IV** undergoes inversion of configuration

(D) The order of reactivity for **I**, **III** and **IV** is : **IV** > **I** > **III**

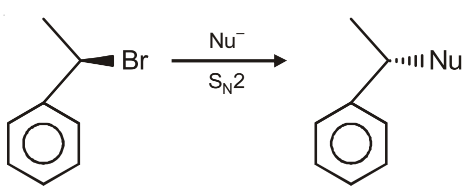
Answer (A, B, C)

Sol. (A) I & II will follow S_N2 when medium is polar aprotic and nucleophile is strong in high concentration.

When medium is highly polar and protic I & III will follow S_N1.

Hence, (B) is correct.

Option (C) is correct as



Inversion in case of S_N2.

(D) is incorrect for both S_N1 and S_N2 conditions.

SECTION - 3 (Maximum Marks : 12)

This section contains **TWO** Paragraphs.

Based on each paragraph, there are **TWO** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble(s) corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 In all other cases

PARAGRAPH 1

Upon heating $KClO_3$ in the presence of catalytic amount of MnO_2, a gas **W** is formed. Excess amount of **W** reacts with white phosphorus to give **X**. The reaction of **X** with pure HNO_3 gives **Y** and **Z**.

33. **Y** and **Z** are, respectively

(A) N_2O_4 and HPO_3

(B) N_2O_5 and HPO_3

(C) N_2O_3 and H_3PO_4

(D) N_2O_4 and H_3PO_3

Answer (B)

34. **W** and **X** are, respectively

(A) O_3 and P_4O_{10}

(B) O_2 and P_4O_6

(C) O_2 and P_4O_{10}

(D) O_3 and P_4O_6

Answer (C)

Solutions of Q.No (33) & (34)

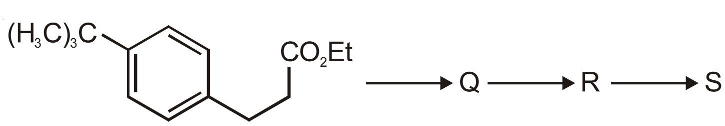
Sol. $2KClO_3 \xrightarrow{[MnO_2]} 2KCl + \underset{(W)}{3O_2}$

$$5O_2 + P_4 \rightarrow \underset{(X)}{P_4O_{10}}$$

$$P_4O_{10} + 4HNO_3 \rightarrow \underset{(X)}{2N_2O_5} + \underset{(Z)}{4HPO_3}$$

PARAGRAPH 2

The reaction of compound **P** with CH_3MgBr (excess) in $(C_2H_5)_2O$ followed by addition of H_2O gives **Q**. The compound **Q** on treatment with H_2SO_4 at 0ºC gives **R**. The reaction of **R** with CH_3COCl in the presence of anhydrous $AlCl_3$ in CH_2Cl_2 followed by treatment with H_2O produces compound **S**. [Et in compound **P** is ethyl group]



P

35. The reactions, **Q** to **R** and **R** to **S**, are

(A) Friedel-Crafts alkylation and Friedel-Crafts acylation

(B) Friedel-Crafts alkylation, dehydration and Friedel-Crafts acylation

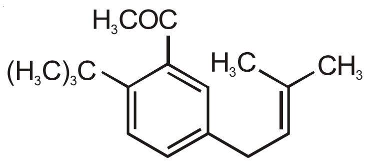
(C) Aromatic sulfonation and Friedel-Crafts acylation

(D) Dehydration and Friedel-Crafts acylation

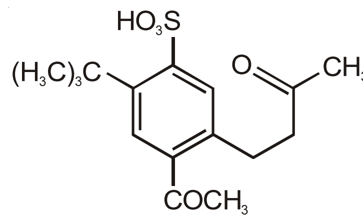
Answer (B)

36. The product **S** is

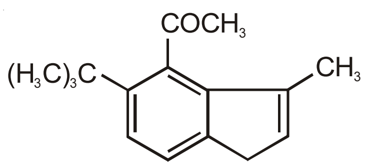
(A)



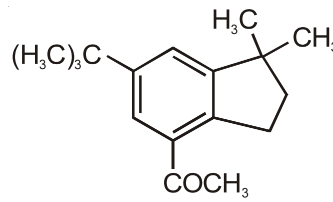
(B)



(C)

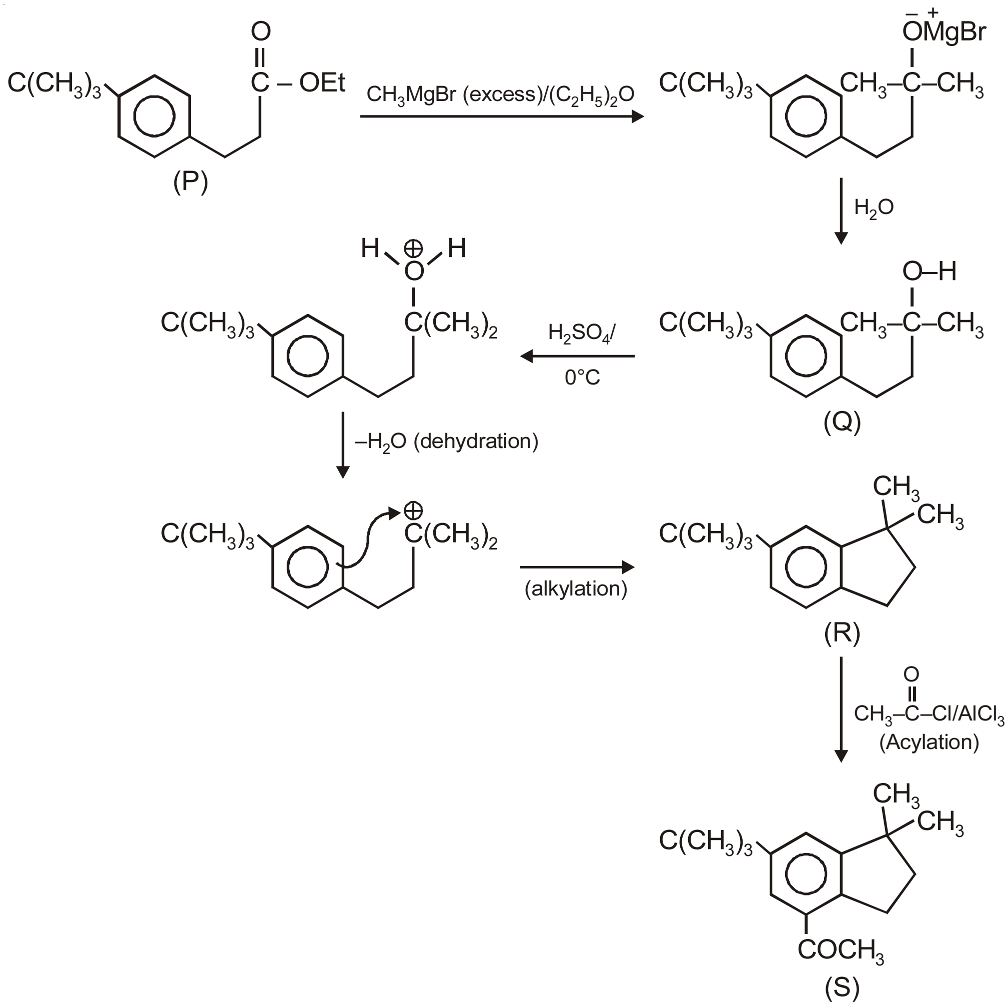


(D)



Answer (D)

Solutions of Q. 35 and 36



MATHS

SECTION - 1 (Maximum Marks : 21)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks : –1 In all other cases

37. Let $S = \{1, 2, 3, ..., 9\}$. For $k = 1, 2, ..., 5$, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

(A) 125 (B) 252

(C) 210 (D) 126

Answer (D)

Sol. Required number of subsets

$= {}^5C_1 \times {}^4C_4 + {}^5C_2 \times {}^4C_3 + {}^5C_3 \times {}^4C_2 + {}^5C_4 \times {}^4C_1 + {}^5C_5 \times {}^4C_0$

$= 5 + 40 + 60 + 20 + 1$

$= 126$

Alternate method

Coefficient of x^5 in $(1 + x)^5(1 + x)^4$

$= {}^9C_5$

$= 126$

38. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$\overrightarrow{OP}\cdot\overrightarrow{OQ}+\overrightarrow{OR}\cdot\overrightarrow{OS}=\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}=\overrightarrow{OQ}\cdot\overrightarrow{OR}+\overrightarrow{OP}\cdot\overrightarrow{OS}$

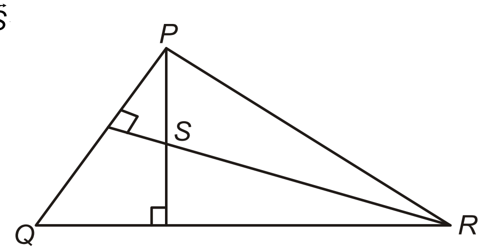
Then the triangle PQR has S as its

(A) Orthocenter (B) Centroid

(C) Circumcentre (D) Incentre

Answer (A)

Sol. $\overrightarrow{OP}\cdot\overrightarrow{OQ}+\overrightarrow{OR}\cdot\overrightarrow{OS}=\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}=\overrightarrow{OQ}\cdot\overrightarrow{OR}+\overrightarrow{OP}\cdot\overrightarrow{OS}$



$\overrightarrow{OP}\cdot\overrightarrow{OQ}+\overrightarrow{OR}\cdot\overrightarrow{OS}=\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}$

$\Rightarrow \overrightarrow{OP}\cdot(\overrightarrow{OQ}-\overrightarrow{OR})+\overrightarrow{OS}\cdot(\overrightarrow{OR}-\overrightarrow{OQ})=0$

$\Rightarrow \overrightarrow{RQ}\cdot(\overrightarrow{OP}-\overrightarrow{OS})=\vec{0}$

$\Rightarrow \overrightarrow{RQ}\cdot\overrightarrow{SP}=\vec{0}$

$\Rightarrow \overrightarrow{RQ}\perp\overrightarrow{SP}$

and similarly from $\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}=\overrightarrow{OQ}\cdot\overrightarrow{OR}+\overrightarrow{OP}\cdot\overrightarrow{OS}$

$\overrightarrow{SR}\perp\overrightarrow{PQ}$

$\therefore$ S is the orthocentre.

39. If $f:\mathbb{R}\to\mathbb{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x\in\mathbb{R}$, and $f\left(\frac{1}{2}\right)=\frac{1}{2}, f(1)=1$, then

(A) $f'(1)\le 0$

(B) $\frac{1}{2}<f'(1)\le 1$

(C) $0<f'(1)\le\frac{1}{2}$

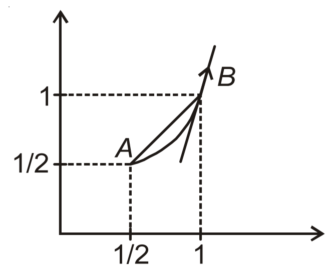
(D) $f'(1)>1$

Answer (D)

Sol. $f''(x)>0$, $f\left(\frac{1}{2}\right)=\frac{1}{2}$ and $f(1) = 1$

$f'(x)$ is always increasing

$$f'(1)>\frac{f(1)-f\left(\frac{1}{2}\right)}{1-\frac{1}{2}}$$



$f'(1)>1$

Slope of tangent at B > Slope of chord AB.

40. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(A) $14x + 2y - 15z = 1$

(B) $14x - 2y + 15z = 27$

(C) $14x + 2y + 15z = 31$

(D) $-14x + 2y + 15z = 3$

Answer (C)

Sol. Required equation of plane is

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix}=0$$

$\Rightarrow$ $-14(x - 1) - 2(y - 1) + (-15)(y - 1) = 0$

$\Rightarrow$ $\boxed{14x+2y+15y=31}$

41. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 5?

(A) 162

(B) 135

(C) 126

(D) 198

Answer (D)

Sol. Let $M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $M^T = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

Sum of diagonal entries = $\sum_{i=1}^{3}\left(a_i^2 + b_i^2 + c_i^2\right) = 5$

$\therefore$ Possible cases are (one 2, one 1 and seven zeros) or (five 1's and four 0's)

$= \frac{9!}{7!} + \frac{9!}{5!4!} = 72 + 126 = 198$

42. If $y = y(x)$ satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx,\ x > 0$ and $y(0) = \sqrt{7}$, then $y(256)$ =

(A) 9

(B) 3

(C) 80

(D) 16

Answer (B)

Sol. As, $dy = \dfrac{dx}{8\sqrt{x}\sqrt{9+\sqrt{x}}\cdot\sqrt{4+\sqrt{9+\sqrt{x}}}}$

Integrating,

$y = \sqrt{4+\sqrt{9+\sqrt{x}}} + c$

at $x = 0$, $y = \sqrt{7} \Rightarrow c = 0$

so, $y = \sqrt{4+\sqrt{9+\sqrt{x}}}$

at $x = 256$, $y = 3$

43. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

(A) $\frac{5}{11}$

(B) $\frac{1}{2}$

(C) $\frac{36}{55}$

(D) $\frac{6}{11}$

Answer (D)

Sol. $x + y + z = 10$

$n(s) = {}^{10+3-1}C_{3-1} = {}^{12}C_2 = \frac{12\times 11}{2} = 66$

Let $z = 2n$, where $n = 0, 1, 2, 3, 4, 5$

$x + y + 2n = 10$

$x + y = 10 - 2n$

Total such solution $= \sum_{n=0}^{5}(11-2n) = 36$

$P(E) = \frac{36}{66} = \frac{6}{11}$

SECTION - 2 (Maximum Marks : 28)

This section contains **SEVEN** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened
Zero Marks	:	0	If none of the bubbles is darkened
Negative Marks	:	–2	In all other cases

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

44. If the line $x = \alpha$ divides the area of region $R = \{(x,y) \in \mathbb{R}^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

(A) $\frac{1}{2} < \alpha < 1$

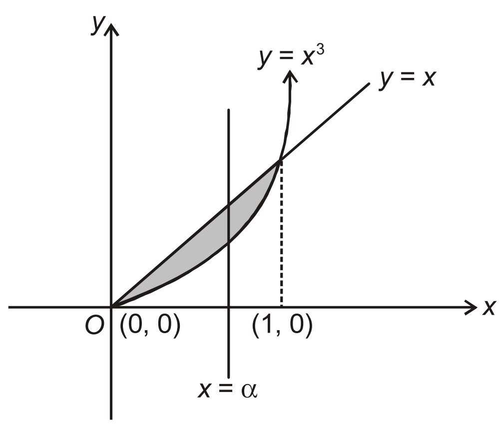
(B) $0 < \alpha \le \frac{1}{2}$

(C) $\alpha^4 + 4\alpha^2 - 1 = 0$

(D) $2\alpha^4 - 4\alpha^2 + 1 = 0$

Answer (A, D)

Sol. $\int_0^\alpha (x - x^3)\,dx = \int_\alpha^1 (x - x^3)\,dx$



$\Rightarrow \left.\frac{x^2}{\alpha} - \frac{x^4}{4}\right|_0^\alpha = \left.\frac{x^2}{2} - \frac{x^4}{4}\right|_\alpha^1$

$\Rightarrow \frac{\alpha^2}{2} - \frac{x^4}{4} = \left(\frac{1}{2} - \frac{1}{4}\right) - \left(\frac{\alpha^2}{2} - \frac{\alpha^4}{4}\right)$

$\Rightarrow \frac{\alpha^4}{2} - \alpha^2 + \frac{1}{4} = 0$

$\Rightarrow \boxed{2\alpha^4 - 4\alpha^2 + 1 = 0}$

Let $f(\alpha) = 2\alpha^4 - 4\alpha^2 + 1$

$f(0) = 1 > 0$, $f\left(\frac{1}{2}\right) = \frac{1}{8} > 0$

$f(1) = -1 < 0$

$\therefore \quad \alpha \in \left(\frac{1}{2}, 1\right)$

45. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt$, then

(A) $g'\left(\frac{\pi}{2}\right) = 2\pi$

(B) $g'\left(\frac{\pi}{2}\right) = -2\pi$

(C) $g'\left(-\frac{\pi}{2}\right) = -2\pi$

(D) $g'\left(-\frac{\pi}{2}\right) = 2\pi$

Answer (No options is correct)

Sol. $g'(x) = \left(\sin^{-1}(\sin 2x)\right) 2\cos 2x - \left(\sin^{-1}(\sin x)\right)\cos x$

$\therefore \quad g'\left(\frac{\pi}{2}\right) = 0,\ g'\left(-\frac{\pi}{2}\right) = 0$

None of the given options is correct.

46. Let $f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|}\cos\left(\frac{1}{1 - x}\right)$ for $x \neq 1$. Then

(A) $\lim_{x\to1^-} f(x)$ does not exist

(B) $\lim_{x\to1^+} f(x)$ does not exist

(C) $\lim_{x\to1^+} f(x) = 0$

(D) $\lim_{x\to1^-} f(x) = 0$

Answer (B, D)

Sol. $f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|}\cos\left(\frac{1}{1 - x}\right)$

$$\lim_{x\to1^+} \frac{1 - x(1 + x - 1)}{(x - 1)}\cos\left(\frac{1}{1 - x}\right)$$

$$\lim_{x\to1^+} \frac{1 - x^2}{(x - 1)}\cos\left(\frac{1}{1 - x}\right)$$

$$\lim_{x\to1^+} -(1 + x)\cos\left(\frac{1}{1 - x}\right)$$ = a number lying between –2 and 2

Hence, limit does not exist.

$$\lim_{x\to 1^-} \frac{1-x(1+(1-x))}{(1-x)}\cos\left(\frac{1}{1-x}\right)$$

$$\lim_{x\to 1^-} \frac{1-x(2-x)}{(1-x)}\cos\left(\frac{1}{1-x}\right)$$

$$\lim_{x\to 1}(1-x)\cos\left(\frac{1}{1-x}\right)=0$$

47. If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x\in\mathbb{R}$, and $f(0) = 1$, then

(A) $f'(x) < e^{2x}$ in $(0, \infty)$

(B) $f(x) > e^{2x}$ in $(0, \infty)$

(C) $f(x)$ is increasing in $(0, \infty)$

(D) $f(x)$ is decreasing in $(0, \infty)$

Answer (B, C)

Sol. $f'(x)-2f(x)>0$

$e^{-2x}\cdot f'(x)-2e^{-2x}\cdot f(x)>0$

$\frac{d}{dx}\left(e^{-2x}f(x)\right)>0 \Rightarrow e^{-2x}\cdot f(x)$ is increasing function.

$e^{-2x}\cdot f(x)>1$ for all $x\in(0, \infty)$

$f(x)>e^{2x}$

$\because \quad f'(x)>2f(x)>e^{2x}>0$

$\therefore \quad f(x)$ is increasing

Also as, $f'(x)=\frac{f(x)-f(0)}{x-0} \Rightarrow f'(x)=\frac{f(x)-1}{x}$

i.e., $f'(x)>e^{2x}\ \forall\ x\in(0, 1)$

$<e^{2x}\ \forall\ x\in(1, \infty)$

48. Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

(A) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(B) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(C) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

(D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

Answer (A, B)

Sol. As 2(cosβ – cosα) = 1 – cosα · cosβ

$$\Rightarrow \cos\alpha = \frac{2\cos\beta - 1}{2-\cos\beta}$$

Using componendo and dividendo

$$\Rightarrow \frac{1-\cos\alpha}{1+\cos\alpha} = 3\left(\frac{1-\cos\beta}{1+\cos\beta}\right)$$

$$\Rightarrow \tan^2\frac{\alpha}{2} - 3\tan^2\frac{\beta}{2} = 0$$

So, $\tan\frac{\alpha}{2} + \sqrt{3}\tan\frac{\beta}{2} = 0$

Or

$$\tan\frac{\alpha}{2} - \sqrt{3}\tan\frac{\beta}{2} = 0$$

49. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$

(B) $f(x)$ attains its minimum at $x = 0$

(C) $f(x)$ attains its maximum at $x = 0$

(D) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

Answer (C, D)

Sol. $C_1 \to C_1 - C_2$

$$f(x) = \begin{vmatrix} 0 & \cos 2x & \\ -2\cos x & \cos x & \\ 0 & \sin x & \cos x \end{vmatrix}$$

$\Rightarrow \quad f(x) = 2\cos x(\cos 2x \cos x - \sin x \sin 2x)$

$\Rightarrow \quad f(x) = 2\cos x \cos 3x$; ($f(0) = 2$ maximum at $x = 0$)

$\Rightarrow \quad f(x) = \cos 4x + \cos 2x$

$f'(x) = -2\sin 2x(4\cos 2x + 1)$

$\Rightarrow \quad \sin 2x = 0$ or $\cos 2x = -\frac{1}{4}$

$2x = 0, \pi, -\pi$

$x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$ and $\cos 2x = -\frac{1}{4}$ gives 4 solutions in $(-\pi, \pi)$

$\therefore$ Total number of solutions = 7

50. If $I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$, then

(A) $I < \frac{49}{50}$

(B) $I > \log_e 99$

(C) $I < \log_e 99$

(D) $I > \frac{49}{50}$

Answer (C, D)

Sol. $I = \sum_{k=1}^{98} (k+1)\left(\int_{k}^{k+1} \frac{dx}{x(x+1)}\right)$

$$= \sum_{k=1}^{98} (k+1)\left\{\log\left(\frac{x}{1+x}\right)\right\}_{k}^{k+1} = \sum_{k=1}^{98} (k+1)\left\{\log\left(\frac{k+1}{k+2}\right) - \log\left(\frac{k}{k+1}\right)\right\}$$

$$= \sum_{k=1}^{98} \left\{(k+1)\log\left(\frac{k+1}{k+2}\right) - k\log\left(\frac{k}{k+1}\right) + \log(k+1) - \log k\right\}$$

$$= \left\{99\log\left(\frac{99}{100}\right) - \log\left(\frac{1}{2}\right)\right\} + (\log 99 - \log 1)$$

$$= 99\log\left(\frac{99}{100}\right) + \log 2 + \log_e(99)$$

SECTION - 3 (Maximum Marks : 12)

This section contains **TWO** Paragraphs.

Based on each paragraph, there are **TWO** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble(s) corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 In all other cases

PARAGRAPH-1

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

51. $|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin(P + R)$

(B) $\sin 2R$

(C) $\sin(P + Q)$

(D) $\sin(Q + R)$

Answer (C)

Sol. $\left|\overrightarrow{OX}\times\overrightarrow{OY}\right| = \dfrac{\left|\overrightarrow{QR}\times\overrightarrow{RP}\right|}{pq}$

$= \dfrac{pq\sin R}{pq}$

$= \sin(P+Q)$

52. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is

(A) $-\dfrac{3}{2}$

(B) $\dfrac{5}{3}$

(C) $-\dfrac{5}{3}$

(D) $\dfrac{3}{2}$

Answer (A)

Sol. $\cos(P + Q) + \cos(Q + R) + \cos(R + P) = -(\cos P + \cos Q + \cos R)$

Maximum value of $\cos P + \cos Q + \cos R = \dfrac{3}{2}$

Hence minimum of $-(\cos P + \cos Q + \cos R) = -\dfrac{3}{2}$

PARAGRAPH-2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a+b\sqrt{5}=0$, then $a = 0 = b$.

53. $a_{12} =$

(A) $2a_{11}+a_{10}$

(B) $a_{11}-a_{10}$

(C) $a_{11}+2a_{10}$

(D) $a_{11}+a_{10}$

Answer (D)

Sol. $\alpha^2 - \alpha - 1 = 0 \quad \Rightarrow \quad \alpha^{12} = \alpha^{11} + \alpha^{10} \quad$ ...(i)

and $\quad \beta^{12} = \beta^{11} + \beta^{10} \quad$ ...(ii)

Multiplying (i) by p and (ii) by q and adding, $a_{12} = a_{11} + a_{10}$

54. If $a_4 = 28$, then $p + 2q =$

(A) 21

(B) 14

(C) 7

(D) 12

Answer (D)

Sol. $a_4 = 28$

$$p\left(\frac{1+\sqrt{5}}{2}\right)^4 + q\left(\frac{1-\sqrt{5}}{2}\right)^4 = 28$$

$$\Rightarrow \quad 56(p+q) + 24\sqrt{5}(p-q) = 28 \times 16$$

$$\Rightarrow \quad p = q = 4$$